LECTURE NOTES

ON

STRENGTH OF MATERIAL(Th.2)

3RD SEMESTER

DEPARTMENT OF MECHANICAL ENGINEERING

GOVERNMENT POLYTECHNIC

SONEPUR-767017

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TH-2 STRENGTH OF MATERIAL

Name of the Course: Diploma in **Mech/Auto/Aero & Other Mechanical Allied Branches**			
Course code:		Semester	3rd
Total Period:	60	Examination	3 hrs
Theory periods:	4 P/W	I.A TEST	20
Maximum marks:	100	End Semester Examination:	80

A. RATIONALE :

Strength of material deals with the internal behaviors of solid bodies under the action of external force. The subject focuses on mechanical properties of material analysis of stress, strain and deformations. Therefore it is an important basic subject of students for Mechanical and Automobile Engg.

B. COURSE OBJECTIVES:

Students will develop ability towards

- Determination of stress, strain under uniaxial loading (due to static or impact load and temperature) in simple and single core composite bars.
- Determination of stress, strain and change in geometrical parameters of cylindrical and spherical shells due to pressure
- Realization of shear stress besides normal stress and computation of resultant stress in two dimensional objects.
- Drawing bending moment and shear force diagram and locating points in a beam where the effect is maximum or minimum.
- Determination of bending stress and torsional shear stress in simple cases
- Understanding of critical load in slender columns thus realizing combined effect of axial and bending load.

C. CHAPTER WISE DISTRIBUTION OF PERIODS

Sl. No.	Topic	Periods
01	Simple Stress & Strain	10
02	Thin cylindrical and spherical shell under internal pressure	08
03	Two dimensional stress systems	10
04	Bending moment& shear force	10
05	Theory of simple bending	10
06	Combined direct & Bending stresses	06
07	Torsion	06
	Total Period:	60

D. COURSE CONTENTS

1.0 Simple stress& strain

- 1.1 Types of load, stresses & strains,(Axial and tangential) Hooke's law, Young's modulus, bulk modulus, modulus of rigidity, Poisson's ratio, derive the relation between three elastic constants,
- 1.2 Principle of super position, stresses in composite section
- 1.3 Temperature stress, determine the temperature stress in composite bar (single core)
- 1.4 Strain energy and resilience, Stress due to gradually applied, suddenly applied and impact load
- 1.5 Simple problems on above.

2.0 Thin cylinder and spherical shell under internal pressure

- 2.1 Definition of hoop and longitudinal stress, strain
- 2.2 Derivation of hoop stress, longitudinal stress, hoop strain, longitudinal strain and volumetric strain
- 2.3 Computation of the change in length, diameter and volume
- 2.4 Simple problems on above

3.0 Two dimensional stress systems

- 3.1 Determination of normal stress, shear stress and resultant stress on oblique plane
- 3.2 Location of principal plane and computation of principal stress
- 3.3 Location of principal plane and computation of principal stress and Maximum shear stress using Mohr's circle

4.0 Bending moment& shear force

- 4.1 Types of beam and load
- 4.2 Concepts of Shear force and bending moment
- 4.3 Shear Force and Bending moment diagram and its salient features illustration in cantilever beam, simply supported beam and over hanging beam under point load and uniformly distributed load

5.0 Theory of simple bending

- 5.1 Assumptions in the theory of bending.
- 5.2 Bending equation, Moment of resistance, Section modulus& neutral axis.
- 5.3 Solve simple problems.

6.0 Combined direct & bending stresses

- 6.1 Define column
- 6.2 Axial load, Eccentric load on column,

6.3 Direct stresses, Bending stresses, Maximum& Minimum stresses
Numerical problems on above
6.4 Buckling load computation using Euler's formula (no derivation) in
Columns with various end conditions

7.0 **Torsion**

7.0 Assumption of pure torsion
7.1 The torsion equation for solid and hollow circular shaft
7.2 Comparison between solid and hollow shaft subjected to pure torsion

Syllabus to be covered up to I.A - Chapters 1, 2, 3&4

Learning resources:

Sl. No.	Author	Title of the book	Publisher
01	S Ramamrutham	Strength of Materials	Dhanpat Rai
02	R.K Rajput	Strength of Materials	S.Chand
03	R.S khurmi	Strength of Materials	S.Chand
04	G.H.Ryder	Strength of Materials	Mc millan and co. lmtd
05	S Timoshenko and D H Young	Strength of Materials	TMH

Chapter-1.0

SIMPLE STRESS AND STRAIN

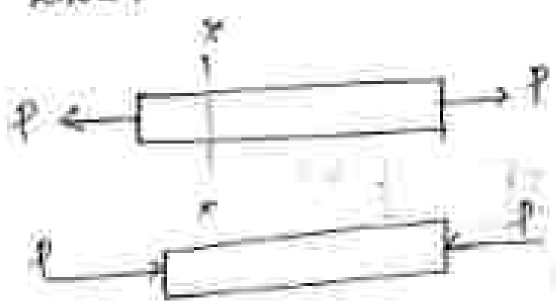
Load:- External force acting on the body is known as load or force. Hydraulic force, steam pressure, tensile force, spring force are diff types of forces. Again load may be classified as live load and dead load.

Live and Dead Load:-

Dead Load:- A constant load in a structure (such as a bridge, building or machine) that is due to the weight of the members, the supporting structure and permanent attachments or accessories.

Types of Load:-

The simplest types of load is direct push or pull, technically known as tension or compression.



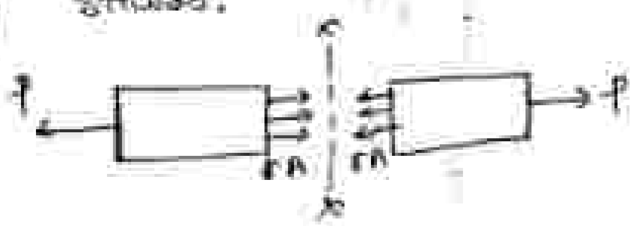
If a member is in motion the load may be caused partially by dynamic or inertia forces. For instance, the CR of a reciprocating engine, load on a flywheel etc.

Introduction :-

When an external force acts on a body, the body tends to undergo some deformation. Due to cohesion b/w the molecules the body resists deformation. The resistance by which the material of the body opposes the deformation is known as strength of material. Within a certain limit (i.e. in the elastic stage) the resistance offered by the material is proportional to the deformation brought out on the material by the external force. Also within the elastic limit, resistance is equal to the external force.

Stress :-

The force transmitted across any section divided by the area of that section is called intensity of stress or simply stress.



$$\boxed{\sigma = \frac{P}{A}}$$

Where σ = Stress
P = Load
A = Area Crosssectional area
σA = Internal force of cohesion

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The force of resistance per unit area, offered by a body against deformation is known as stress. The load is applied on the body while the stress is induced in the material of the body.

Units of Stress:

MKS, $\sigma = \frac{P}{A} \left(\frac{kgf}{m^2}\right), \left(\frac{kgf}{cm^2}\right)$

SI, $\sigma = \frac{P}{A} \left(\frac{N}{m^2}\right), \left(\frac{N}{cm^2}\right), \left(\frac{N}{mm^2}\right)$

Larger units

Kilo $= 10^3$, Mega $= 10^6$, Giga $= 10^9$, Terra $= 10^{12}$

Smaller units

Milli $= 10^{-3}$, Micro $= 10^{-6}$, Nano $= 10^{-9}$, Pico $= 10^{-12}$

Note. 1. Newton is the force acting on a mass of one kg and produces an acceleration of $1\,m/s^2$.

i.e. $1N = 1(kg) \times 1\,m/s^2$

2. 1 Pascal $= 1\,Pa = 1\,N/m^2$

Strain :- It is a measure of the deformation produced in the member by the load. When a body is subjected to some external force, there is some change in dimension of the body. The ratio of change of body to the original dimension is known as strain. Strain is dimensionless.

Mathematically, $e = \frac{\text{change in dimen.}}{\text{original dimen}} = \frac{\delta l}{l}$

Strain may be :

1- Tensile strain   2. Compressive strain

3- Volumetric strain   4. Shear strain

Tensile strain :- If there is some increase in dimension of a body due to external force, then the ratio of increase in dimension to the original dimension is known as Tensile strain.

Compressive strain :-
decrease in dimension due to external load

Volumetric strain = $\frac{\text{change in volume}}{\text{original volume}}$

Shear strain = strain due to shear force.

Note:- Internal resistance force offered by the body against deformation is known as strength of a material. It means stress are developed when a strain is restrained. If strain is free to occur then stress will not develop in that direction. That strain is the cause of stress.

Types of stresses:-

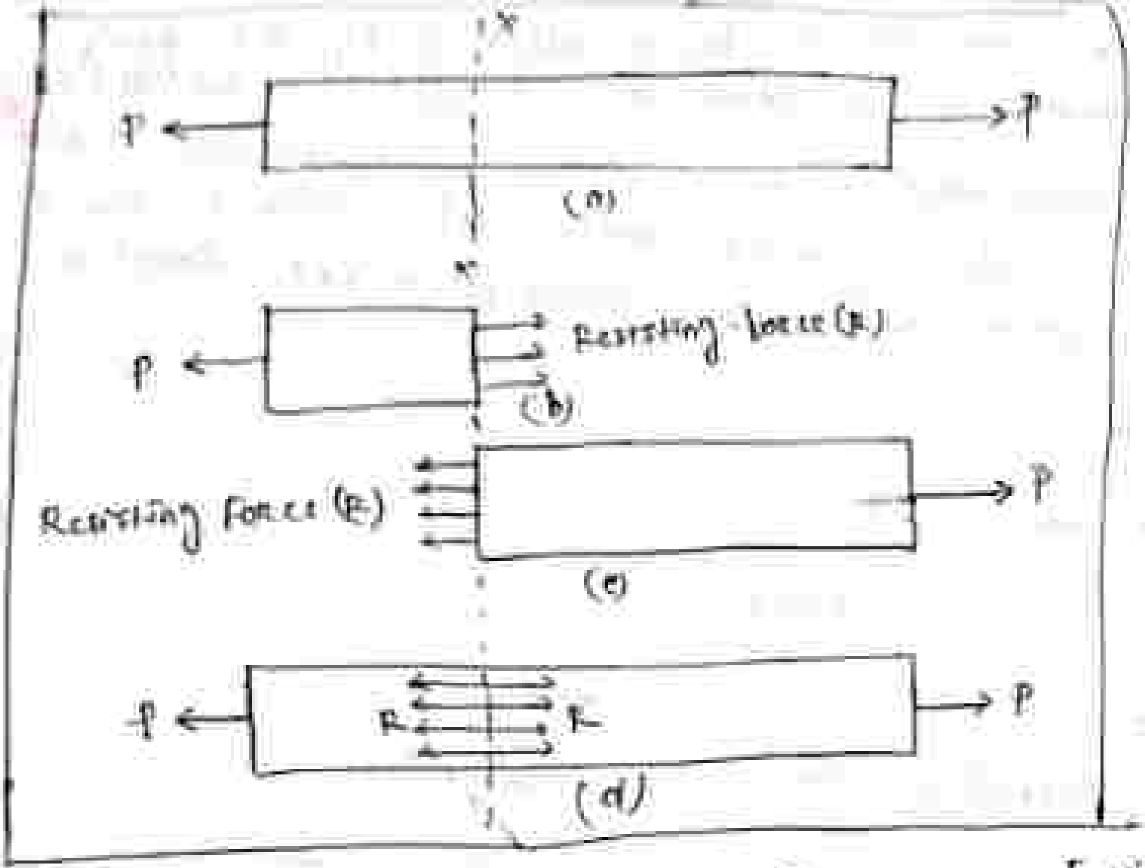
- stress
  - Normal stress (Tensile/comp.)
    - Direct normal stress
    - Bending stress
  - shear stress
    - Direct shear stress
    - Torsional shear stress

* Normal stresses Tensile (comp.)

Normal stress is the stress which acts in a direction perpendicular to the area.

* Tensile normal stress:-

The stress induced in a body, when subjected to two equal and opposite pulls as a result of which there is an increase in length, is known as Tensile stress. The tensile stress acts normal to the area and it pulls on the area.

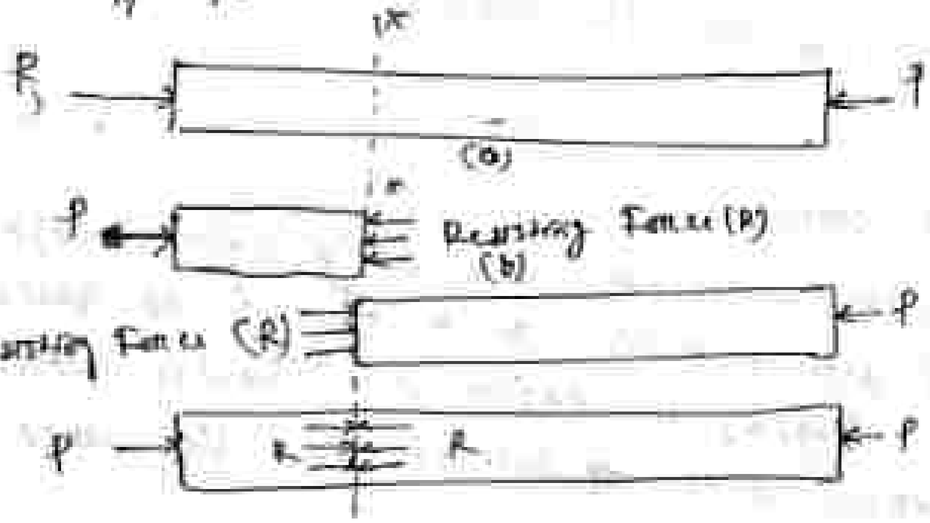


$\therefore$ Tensile stress (σ) $= \dfrac{\text{Resisting Force (R)}}{\text{Crosssectional Area (A)}} = \dfrac{\text{Tensile load (P)}}{A}$

Tensile strain (e) $= \dfrac{\text{Increase in length}}{\text{Original length}} = \dfrac{\delta L}{L}$

* Normal stress (Compressive)

two equal and opposite pushes / decrease in length b/c of pushes on the area.



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Shear Stress :- (τ)

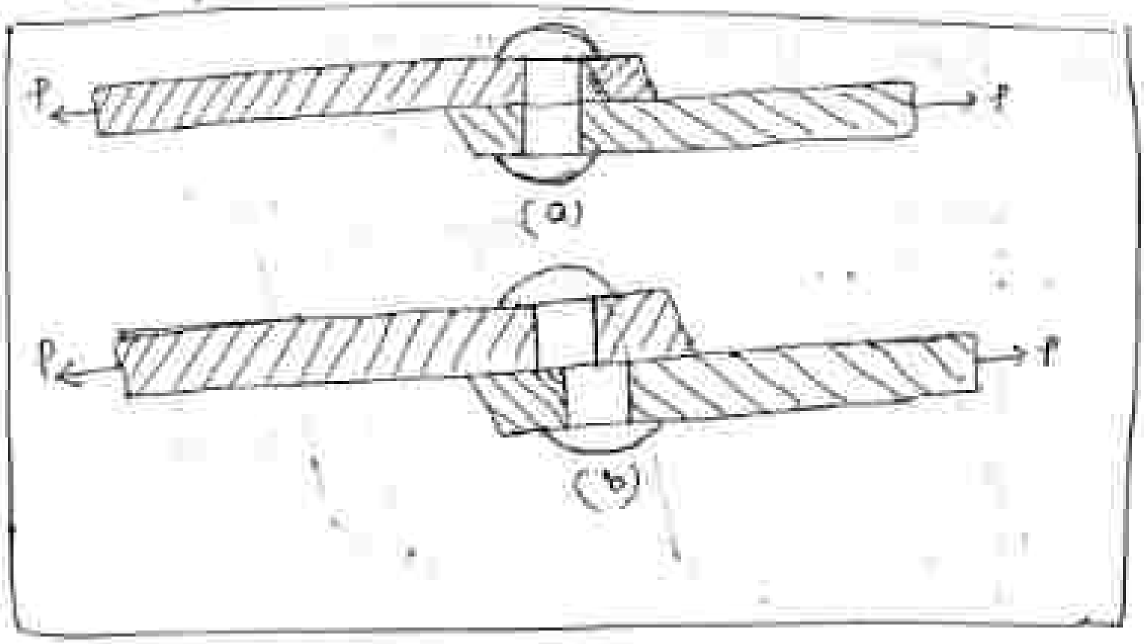
Shear stress is a stress state where the stress is parallel to the surface of the material, as opposed to normal stress when the stress is vertical to the surface.

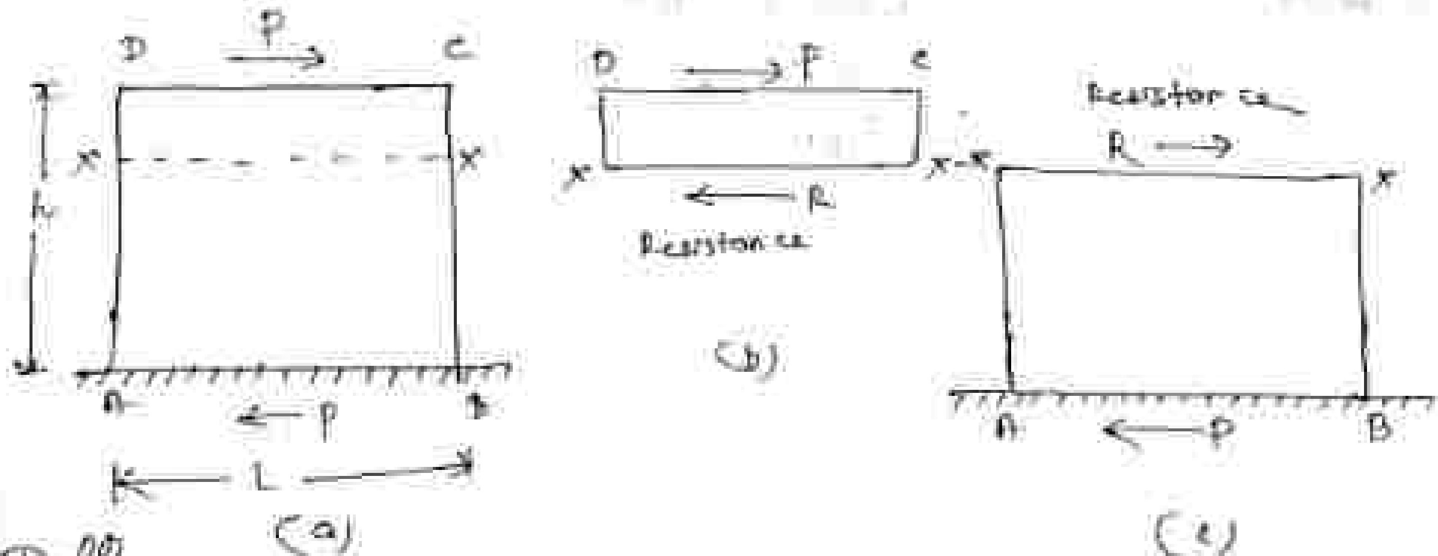
Shear stress is relevant to the motion of fluids upon surfaces which result in the generation of shear stress.

Also constructions of in soil can fail due to shear.

e.g., The weight of an earth-filled dam may cause the sub-soil to collapse, like a small landslide.

* road destroyed by shear.





Defn

The external force acting on an surface or object parallel to the slope or plane in which it lies; the stress tending to produce shear.

The upper part will be in equilibrium if P = Resistance

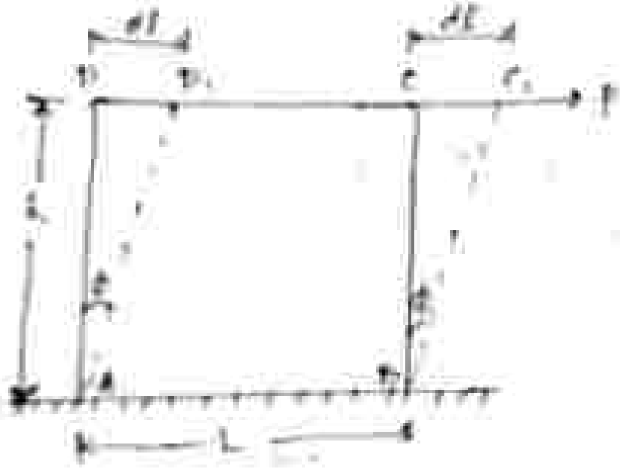
Hence — $P = R$

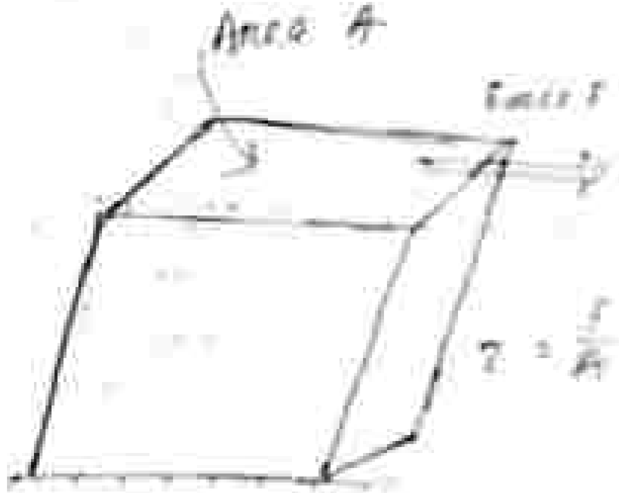
This resistance is known as shear resistance. And the shear resistance per unit area is known as shear stress which is represented by τ.

$$\text{Shear stress}, \tau = \frac{\text{shear resistance}}{\text{shear area}} = \frac{R}{A}$$

$$= \frac{P}{L \times 1} \quad (\because R = P \text{ \& } L = \text{area}, \text{ width} = 1 \text{ unit})$$

As the bottom of the block is fixed,





$$\text{Shear strain } (\phi) = \frac{\text{Transverse displacement}}{\text{distance of AD}}$$

$$= \frac{DD_1}{AD} = \frac{dl}{h}$$

Elasticity and Elastic Limit

It is the property of a material by virtue of which after removal of load specimen regains its original dimension within the elastic limit. The curve may be linear or non linear.

Elastic limit is the limit

Hooke's Law And Elastic Moduli

Assumptions

① Material is homogeneous (properties of metal are same at all pts)
② " " Isotropic (properties same in all directions)
③ " " Elastic

Acc. to Hookes law, within elastic limit stress is directly proportional to strain. This means the ratio of stress to the corresponding strain is a constant within elastic limit. This constant is known as Modulus of Elasticity or Modulus of Rigidity or Elastic Moduli.

Mathematically, $\sigma \propto \epsilon$

$\boxed{\sigma = E\epsilon}$ or $\boxed{\frac{\sigma}{\epsilon} = E}$

Where E = Youngs Modulus or modulus of elasticity

Modulus of Rigidity or Shear Modulus

The ratio of shear stress to the corresponding shear strain ~~is known as~~ within elastic limit is known as Shear Modulus. It is also known as Modulus of Rigidity or denoted by C or G or N.

$$G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$$

Factor of Safety :-

Ratio of ultimate tensile stress to the working (or permissible) stress.

Mathematically, Factor of safety (f) = $\dfrac{\text{Ultimate stress}}{\text{Permissible stress}}$

Ultimate stress.

Stresses in Composite Section

Any tensile or comp. member which consists of two or more bars or tubes in parallel usually of different materials is called compound bars.

Analysis:-

A compound bar is made up of a rod of area A_1 and modulus E_1 and a tube of area A_2 and modulus E_2. If a compressive load (P) is applied to the compound bar find how the load is shared. Since the rod and tube are of the same initial length and must remain together then the strain in each part must be same. The total load carried is P & let load shared is W_1 & W_2.

We have $e_1 = e_2$ & $l_1 = l_2$

$\sigma = E\varepsilon$, $\varepsilon = \frac{\sigma}{E}$, $\frac{\delta l}{l} = \frac{\sigma}{E}$, $\boxed{\varepsilon = \frac{\sigma}{E}}$

Compatibility equation: $\frac{W_1}{A_1 E_1} = \frac{W_2}{A_2 E_2}$

Equilibrium equation: $W_1 + W_2 = P$ —(1)

Substituting $W_2 = \frac{A_2 E_2}{A_1 E_1} \times W_1$ in eqn (1)

$e_1 = e_2$

$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$

$\boxed{\sigma_1 = \frac{E_1}{E_2}\sigma_2}$

$\boxed{\sigma_2 = \frac{E_2}{E_1}\sigma_1}$

$$W_1 + W_1 \frac{A_2 E_2}{A_1 E_1} = P$$

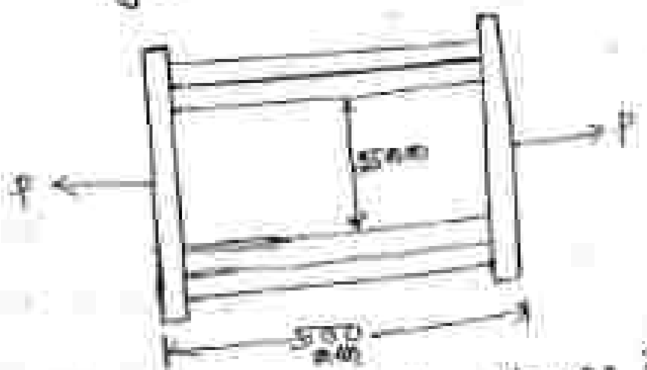
$$\Rightarrow W_1\left(1 + \frac{A_2 E_2}{A_1 E_1}\right) = P \quad \text{or}$$

$$\boxed{W_1 = \frac{P A_1 E_1}{A_1 E_1 + A_2 E_2}} \quad \text{then} \quad \boxed{W_2 = \frac{P A_2 E_2}{A_1 E_1 + A_2 E_2}}$$

$\boxed{P = \sigma_2 A_2 + \sigma_1 A_1}$

Example :-

A composite bar is made up of a brass rod of 25mm diameter enclosed in a steel tube being co-axial of 40mm external diameter and 30mm internal diameters as shown below. They are securely fixed at each end. If the stress in the brass and steel are not to exceed 70MP and 100MPa respectively, find the load (P) the composite bar can safely carry.



Also find the change in length, if the composite bar is 500mm long. Take E for steel tube as 200 GPa and brass rod as 80 GPa respectively.

Sol^n:- Given Data

Let steel tube denotes 1 & brass rod denotes as 2.

Steel

$d_{1(o)} = 40mm$

$d_{1(i)} = 30mm$

$E_1 = 200\ GPa$

$\sigma_1 = 100\ MPa$

$W_1 =$ load carried by tube

brass rod

$d_2 = 25mm$

$E_2 = 80\ GPa$

$\sigma_2 = 70\ MPa$

$W_2 =$ load carried by rod

$A_1 = \frac{\pi}{4}(d_o^2 - d_i^2)$

$= \frac{\pi}{4}(40^2 - 30^2) = 550mm^2$

$A_2 = \frac{\pi}{4}(d_2)^2$

$= \frac{\pi}{4}(25^2)$

$= 491\ mm^2$

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From compatible equation:

$$\boxed{\frac{W_1}{A_1 E_1} = \frac{W_2}{A_2 E_2}}$$

$$\Rightarrow W_1 = W_2 \times \frac{A_1 E_1}{A_2 E_2} = W_2 \times \frac{550 \times 260}{491 \times 80}$$

$$\Rightarrow \boxed{W_1 = 2.8 W_2}$$

stress $= \frac{P}{A}$

$W_1 = \sigma_1 A_1 = 120 \times 550 = 66000 N$

$\sigma = E \epsilon$

& $W_2 = \frac{W_1}{2.8} = \frac{66000}{2.8} = 2357 N$

$\epsilon = \frac{\sigma}{E} = \frac{P}{AE}$

From equilibrium equation,

$$\boxed{W_1 + W_2 = P}$$

$\Rightarrow P = 66000 + 2357 = 89.57 kN$ (Ans)

change in length,

$$\boxed{\delta l_1 = \delta l_2} = \frac{W_1 l_1}{A_1 E_1} = \frac{66000 \times 500}{550 \times 220 \times 10^3} = 0.3 mm \text{ (A)}$$

Poissons Ratio :-

The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poissons Ratio & denoted by μ

$$\mu = \left| \frac{\text{Lateral strain}}{\text{Longitudinal strain}} \right|$$

As lateral strain is opposite in sign to the longitudinal strain, hence

Lateral strain = $-\mu \times$ Longitudinal strain

μ varies b/w 0.1 to 0.35

Rubber = 0.45 - 0.50

μ value

Cork = 0
Glass = 0.05
Concrete = 0.1 to 0.2
Elastic materials = 0.25 - 0.42
Rubber & perfectly plastic material = 0.5 (approx.)

Longitudinal Strain:-

When a body is subjected to an axial tensile load, there is an increase in length of the body. But at the same time there is a decrease in other dimensions of the body at right angle to the line of action of the applied load. Then the body is having axial deformation and also deformation at right angles to the line of action of the applied load (i.e lateral deformation).

Longitudinal strain The ratio of axial deformation to original length of the body is known as longitudinal (or linear) strain. It is also defined as the deformation of the body per unit length in the direction of the applied load.

Let L = Original length of the body.
P = Tensile load acting on the body.
δL = ,

$$\text{Longitudinal strain} = \frac{\delta L}{L}$$

Diagonal strain.

Lateral Strain :-

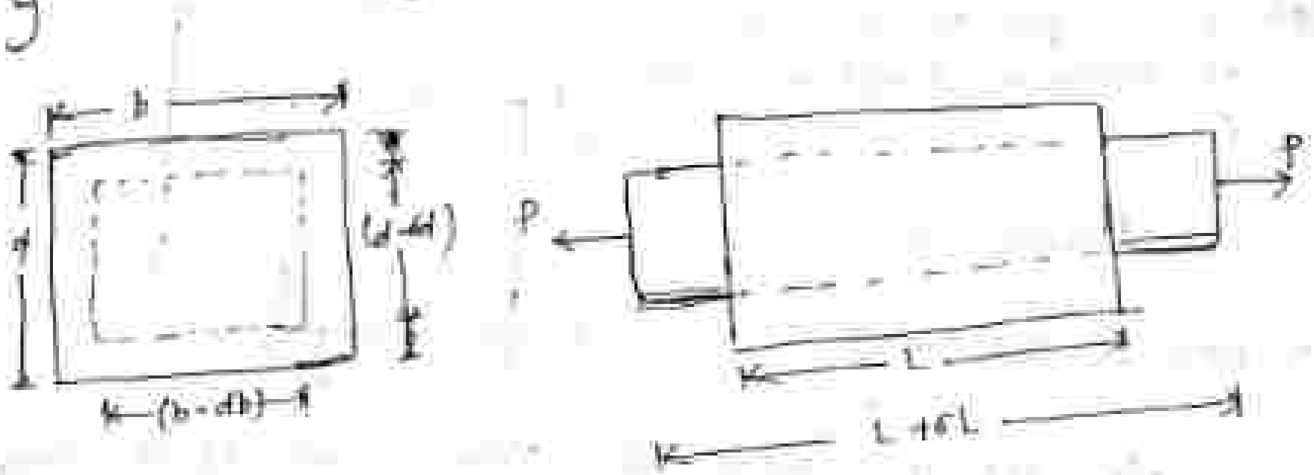
The strain at right angle to the direction of applied load is known as lateral strain.

Let, L = length of rectangular bar
b = breadth, d = depth is subjected to tensile load P. Then length of the bar will increase & breadth and depth will decrease.

Let, δL = Increase in length
δb = Decrease in breadth
δd = " " depth

Then Longitudinal strain = $\frac{\delta L}{L}$

Lateral strain = $\frac{\delta b}{b}$ or $\frac{\delta d}{d}$



Volumetric strain :-

The ratio of change in volume to the original volume of a body (when a body is subjected to a single force or a system of forces) is called volumetric strain.

Mathematically, $e_v = \frac{\delta V}{V}$

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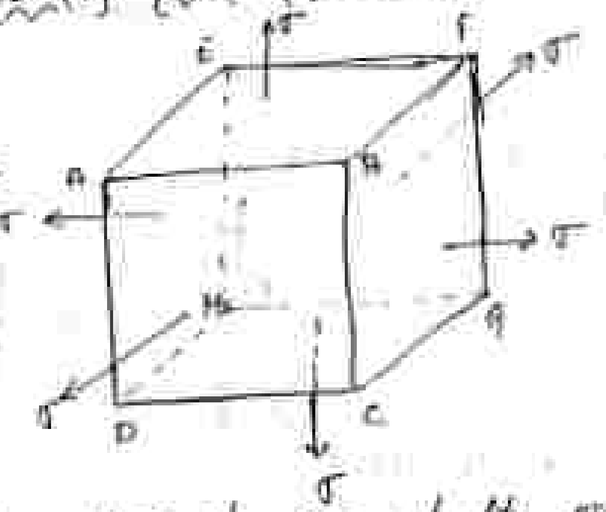
Bulk modulus :-

When a body is subjected to the mutually in like and equal direct stresses, the ratio of direct in to the corresponding volumetric strain is found to be constant for a given material when the deformation is within a certain limit. The ratio is known as Bulk modulus. It is usually denoted by K.

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\left(\frac{dV}{V}\right)}$$

Relationship b/w Youngs modulus(E) & Bulk modulus(K)



Let L = length of the cube

dL = change in length of the cube

E =

σ =

μ = Poissons ratio

Volume of the cube, $V = L^3$.

Now let us consider the strain of one of the sides of the cube (say AB) under the action of three mutually in stresses. This side will suffer the following three strains:

1. strain of AB due to the faces AEHD and BFGC. This strain is tensile & is equal to $\frac{\sigma}{E}$.

2. strain of AB due to stress on the faces AEFB and DHG. This is compressive lateral strain and is equal to $-\mu \frac{\sigma}{E}$

3. Strain of AB due to stresses on the faces AB, CD and EFGH. This is also compressive lateral strain and is equal to $-\mu \frac{\sigma}{E}$.

Hence the total strain of AB is given by,

$$\frac{dL}{L} = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E} = \frac{\sigma}{E}(1-2\mu) \text{ —— (i)}$$

Now original volume of the cube, $V = L^3$ —— (ii)

If dL is the change in Length, ~~then~~ then dV is the change in volume

Differentiating eq^n (ii) w.r.t. L

$$\frac{dV}{dL} = \frac{d}{dL}(L^3)$$

$$\Rightarrow dV = 3L^2 \times dL \text{ —— (iii)}$$

Dividing eq^n (iii) by eq^n (ii), we get.

$$\frac{dV}{V} = \frac{3L^2 \times dL}{L^3} = \frac{3dL}{L}$$

Substituting the value of $\frac{dL}{L}$ from eq^n (i) in the above eq^n.

$$\frac{dV}{V} = \frac{3\sigma}{E}(1-2\mu)$$

We have, $K = \frac{\sigma}{\left(\frac{dV}{V}\right)} = \frac{\cancel{\sigma}}{\frac{3\cancel{\sigma}}{E}(1-2\mu)} = \frac{E}{3(1-2\mu)}$

or $\boxed{E = 3K(1-2\mu)}$

~~Relationship b/w modulus of elasticity and modulus of rigidity~~

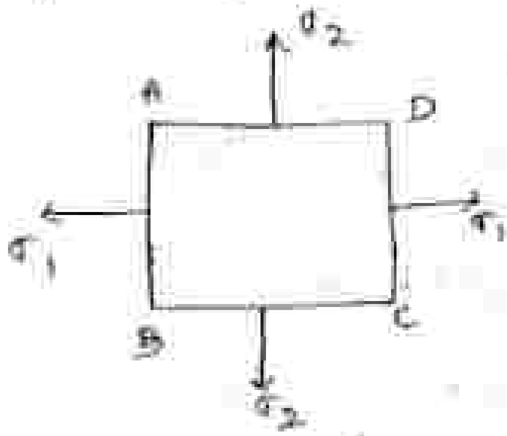
Relationship between stress and strain

* For one-dimensional stress system :

Hooke's Law

* For two-dimensional stress system

1. Longitudinal strain
2. Lateral strain
3. Relationship b/w stress & strain



Consider a 2-D figure ABCD, subjected to two mutually $\perp r$ stresses σ_1 & σ_2.

Let, σ_1 = Normal stress in x-direction

σ_2 = Normal stress in y-direction

Consider the strain produced by σ_1

The stress σ_1 will produce the strain in x as well as y-direction. The strain in the x-direction will be longitudinal & equal to $\frac{\sigma_1}{E}$ whereas the strain in the direction of y is Lateral strain and will be equal to $-\mu \times \frac{\sigma_1}{E}$

Now consider the strain produced by σ_2

strain in y-direction $= \frac{\sigma_2}{E}$

" " x-direction $= -\mu \frac{\sigma_2}{E}$

Let e_1 is the total strain in x-direction

e_2 " " " y-direction

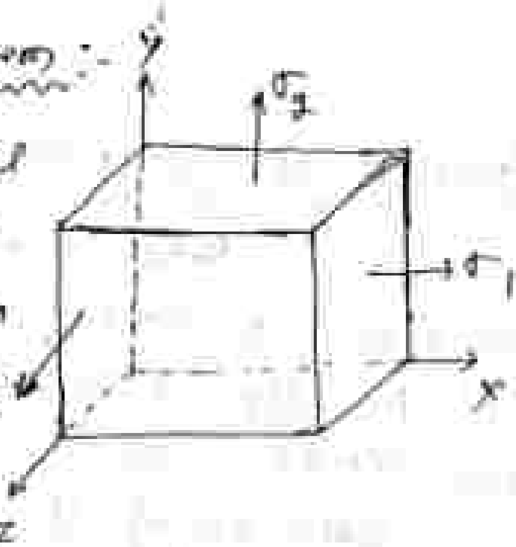
Now total strain in the x-direction due to stresses σ_1 and σ_2 $= \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$

Similarly total strain in the y-direction due to stresses σ_1 & σ_2 $= \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$

Tensile stress +ve
comp. " " -ve

$$\therefore \boxed{\begin{aligned} e_1 &= \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \\ e_2 &= \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E} \end{aligned}}$$

*For three-dimensional stress system:-



Consider a three-dimensional body subjected to 3 orthogonal normal stresses $\sigma_1, \sigma_2, \sigma_3$ acting in the direction of x, y & z respectively.

The stress σ_1 will produce strain in the direction of x and strain in the direction of y and z.
also in the direction of x will be $\frac{\sigma_1}{E}$
the strain in the direction of y and z
whereas the strain in the direction of y and z will be $-\mu \frac{\sigma_1}{E}$

Similarly $\sigma_2 \rightarrow \frac{\sigma_2}{E}$ & $-\mu \frac{\sigma_2}{E}$ (x, z)

Total strain in the direction of x due to stress σ_1, σ_2 and σ_3 $= \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E}$

y-direction $= \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E}$

z-direction $= \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$

Let e_1, e_2 and e_3 are total strain in the direction of x, y and z respectively.

Then,

$$\boxed{\begin{aligned} e_1 &= \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} \\ e_2 &= \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} \\ e_3 &= \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \end{aligned}}$$

Principle of superposition :-

When a number of loads are acting on a body the resulting strain, according to principle of superposition will be the algebric sum of strains caused by individual loads.

While using this principle for an elastic body which is subjected to a number of direct forces (tensile or comp.) at different section along the length of the body, first the free body diagram of individual section is drawn. Then the deformation of each section is obtained. The total deformation of the body will be then equal to the algebric sum of deformations of the individual section.

~~Relationship b/w Young's Modulus & Bulk modulus~~

~~cube subjected to 3 mutually ⊥ Tensile stresses of equal stress intensity~~

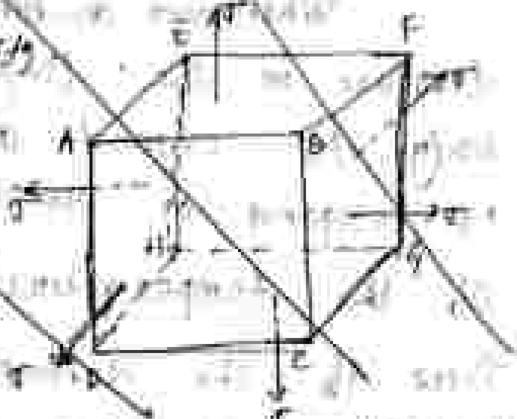
~~Let L = length of cube~~

~~dL =~~

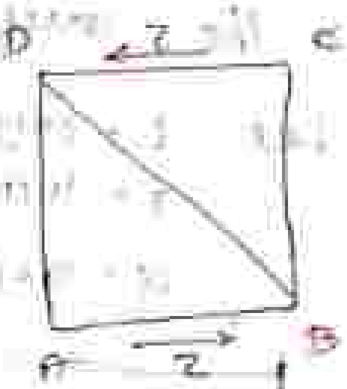
~~σ =~~

~~μ =~~

~~Then volume of the cube, $V = L^3$~~



Relationship b/w Modulus of Elasticity and Modulus of Rigidity



Total tensile strain along diagonal $BD = \frac{\tau}{E}(1+\mu)$ ——(i)

Also we have tensile strain in diagonal $BD = \frac{1}{2} \times$ shear strain

$= \frac{1}{2} \times \frac{\text{shear stress}}{G}$ $\left(G = \frac{SS}{\text{strain}}\right)$

$= \frac{1}{2} \times \frac{\tau}{G}$ ——(ii)

Equating the two equations

$$\frac{\tau}{E}(1+\mu) = \frac{1}{2} \times \frac{\tau}{G}$$

$$\frac{2G}{E} = \frac{1}{(1+\mu)}$$

$$\Rightarrow \boxed{E = 2G(1+\mu)}$$

Thermal stress :-

Whenever ~~a body~~ there is some increase or decrease in the temperature of a body, it cause the body to expand or contract. If the body is allowed to expand or contract freely, with the rise or fall of the temperature, no stress are induced in the body. But if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are called thermal stress or temperature stresses. The corresponding strain is called temp. stresses.

Let L = original length of the body

T = increase in temperature of the body

α = coefficient of linear expansion

(simple bar)

Increase in length due to increase in temp.

$$\delta L = L\alpha T$$

If the ends of the bar fixed to rigid supports, so that its expansion is prevented, then compressive strain induced in the bar.

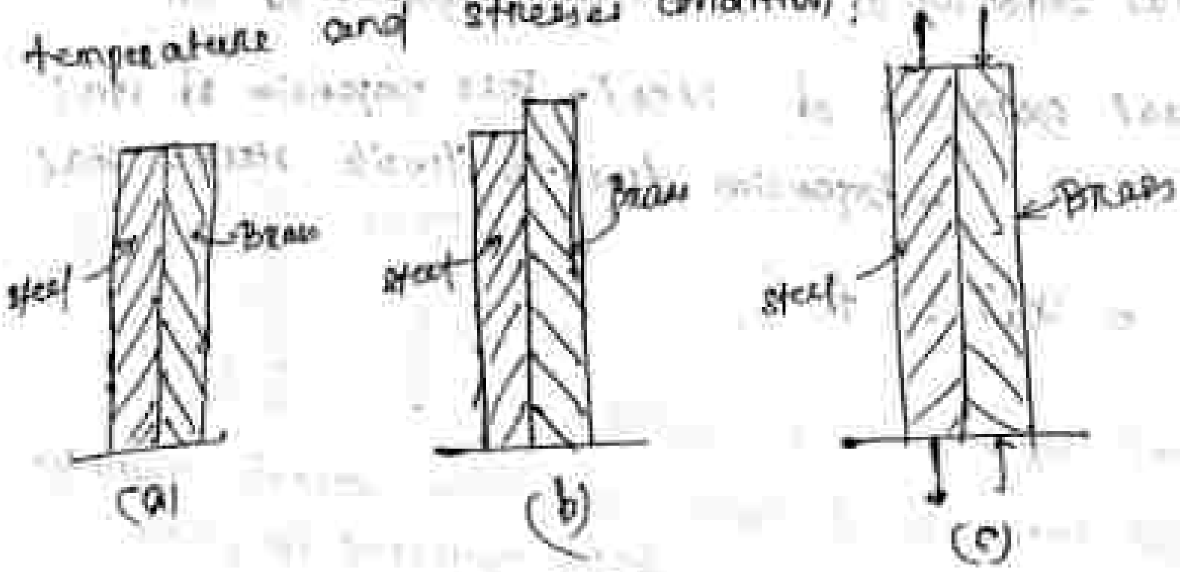
Now expansion registered = $L\alpha T$

strain resisted = $\frac{L\alpha T}{L} = \alpha T$

$$\boxed{\therefore \text{stress} = E(\text{strain}) = \alpha T E}$$

Temperature stress in Composite Bar

If a compound bar made up of several materials is subjected to a change in temperature, there will be a tendency for the components part to expand different amounts due to unequal coefficient of thermal expansion. If the parts are constrained to remain together then actual change in length must be the same for each. This change is the resultant of the effect due to temperature and stresses condition.



Let σ_b = stress in brass
e_b = strain in brass
α_b = Coefficient of linear expansion for brass
A_b = Cross-sectional area of brass bar

$\sigma_s, e_s, \alpha_s, A_s$ = corresponding value for steel, and per unit length.

e = Actual strain of the composite bar

As the comp. load on brass = tensile load on steel.

$\therefore \boxed{\sigma_b A_b = \sigma_s A_s}$

~~Now strain in brass $e_b = \alpha_b T - e$~~ —— (i)

~~" steel $e_s = e - \alpha_s T$~~ —— (ii)

~~Adding eqn (i) and (ii) we get,~~

~~$\alpha_1 + e_2 = \alpha_1 T - \not{f} + \not{E} - \alpha_2 T$~~

~~$= \not{f}\,(\alpha_1 - \alpha_2)$~~

Both the members are not free to expand, and hence the expansion of the composite bar, as a whole will be less than that of brass but more than that of the steel. Hence stress in brass comp. &

Actual expansion of steel = Actual expansion of brass.

But actual expansion of steel = free expansion of steel + expansion due to tensile stress

$= \alpha_s T\,L + \frac{\sigma_s}{E_s} L$

strain $= \frac{\Delta L}{L}$

$\frac{\sigma}{E} = \frac{\Delta L}{L}$

$\Rightarrow \boxed{\Delta L = \frac{\sigma}{E} \times L}$

and Actual expansion of copper = free expansion of copper − contraction due to comp. stress induced in brass

$= \alpha_b T\,L - \frac{\sigma_b}{E_b} L$

Substituting these two values in eqn (i),

we get $\alpha_s \times T \times L + \frac{\sigma_s}{E_s} \times L = \alpha_b \times T \times L - \frac{\sigma_b}{E_b} \times L$

$$\boxed{\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_b \times T - \frac{\sigma_b}{E_b}}$$

where T is the rise of temperature

Stress and strain when the supports yield

If the supports yield by an amount equal to δ, then the actual expansion

$= \text{Expansion due to rise in temp} - \delta$

$= \alpha T L - \delta$

$\therefore$ Actual strain $= \dfrac{\text{Actual expansion}}{\text{Original length}} = \dfrac{(\alpha T L - \delta)}{L}$

And actual stress $=$ Actual strain $\times E$

$$= \frac{(\alpha T L - \delta)}{L} \times E$$

or Actual strain $= \dfrac{\text{Actual stress}}{E} \times L$

$$= \frac{\sigma}{E} \times L$$

Strain energy, resilience stress due to gradually applied load, suddenly apply load and impact load

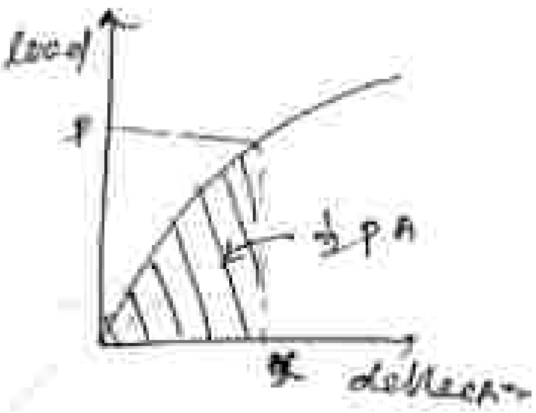
When a body is strained, the energy is absorbed in the body. The energy which is absorbed in the body due to straining effect is known as strain energy. The straining effect may be due to gradually applied load, or suddenly apply load or impact load. Hence the strain energy will be stored in the body when the load is applied gradually or suddenly or with an impact. The strain energy stored in the body is equal to the work done by the apply load in stretching the body.

Resilience :-

The total elastic strain energy stored in a body is commonly known as resilience. Whenever the straining force is removed from the strain body, the body is capable of doing work.

or

It is the total elastic strain energy which can be released on unloading in a given volume of metal. In other words it is the area under load deflection curve upto elastic limit.



*Proof Resilience.

The maximum strain energy stored in a body is known as proof resilience. The strain energy stored in the body will be maximum when the body is stressed upto elastic limit. Hence the proof resilience is the quantity of strain energy stored in a body when strained upto elastic limit.

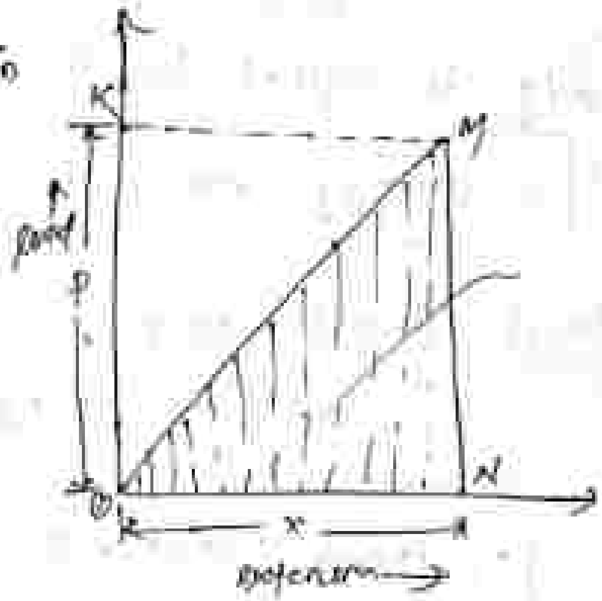
*Modulus of Resilience :-

It is defined as the proof resilience of a material per unit volume. It is an important property of a material. Mathematically,

$$\text{Modulus of Resilience} = \frac{\text{Proof Resilience}}{\text{Volume of the body}}$$

*Expression for strain energy stored in a body when the load is applied gradually :-

The load P performs work in stretching the body. This work will be stored in the body as strained energy which is recoverable after the load P is removed.



Let P = Gradually apply load

x = Extension of the body

A = Cross-sectional area of the body

L = Length of the body

V = Volume of the body

E = Young's modulus

U = strain energy stored in the body &

σ = stress induced in the body

Now work done by the load = Area under load-deflection curve

$= \text{Area of triangle } OXM$

$= \frac{1}{2} \times P \times x$ ——————— (i)

But load, P = stress × Area $= \sigma \times A$

and extension/deflection, x = strain × length

$\left(\because \text{strain} = \frac{\text{Extension}}{\text{Length}} \right)$

$\therefore x = \frac{\text{stress}}{E} \times L = \frac{\sigma}{E} \times L$

Substituting the value of P and x in equation (i), we get

Work done by load $= \frac{1}{2} \times \sigma \times A \times \frac{\sigma}{E} \times L = \frac{1}{2} \frac{\sigma^2}{E} \times A \times L$

$= \frac{\sigma^2}{2E} \times V$ ← volume

But work done by the load in stretching the body is equal to the strain energy stored in the body

$$\boxed{\therefore \text{Energy stored in the body, } U = \frac{\sigma^2}{2E} \times V}$$

Proof Resilience :-

strain energy per unit volume

$$\boxed{\therefore \text{Proof resilience} = \frac{\sigma^2}{2E}}$$

Expression for strain energy stored in a body when the load is applied suddenly

When the load is applied suddenly to a body, the load is constant throughout the process of the deformation of the body.

Consider a bar subjected to a sudden load.

Let P = Load applied suddenly

l = Length of the bar

A = Area of the cross-section

V = Volume of the bar $= A \times l$

E = Young's modulus

x = Extension of the bar

σ = Stress induced by the suddenly applied load P

U = strain energy stored.

As the load is applied suddenly, the load P is constant when the extension of the bar takes place.

$\therefore$ Workdone by load = Load $\times$ Extension $= P \times x$

The maximum strain energy stored (within elastic limit) in a body is given by

$$\boxed{\begin{aligned} U &= \frac{\sigma^2}{2E} \times \text{Volume of the body} \\ U &= \frac{\sigma^2}{2E} \times A \times l \end{aligned}}$$

Equating strain energy stored in the body to the work done, we get

$$\frac{\sigma^2}{2E} \times A \times l = P \times x = P \times \frac{\sigma}{E} \times l$$

$$\frac{\sigma \times A}{2} = P$$

or $$\boxed{\sigma = 2 \times \frac{P}{A}}$$

Maximum stress induced due to suddenly applied load is twice the stress induced when the same load is applied gradually.

After obtaining the value of (σ), and the value of extension and the strain energy stored in the body may be calculated easily.

* Expression for strain energy stored in a body when the load is applied with impact :-

Let P = load dropped

L = Length of the rod

A = Cross-sectional area of the rod

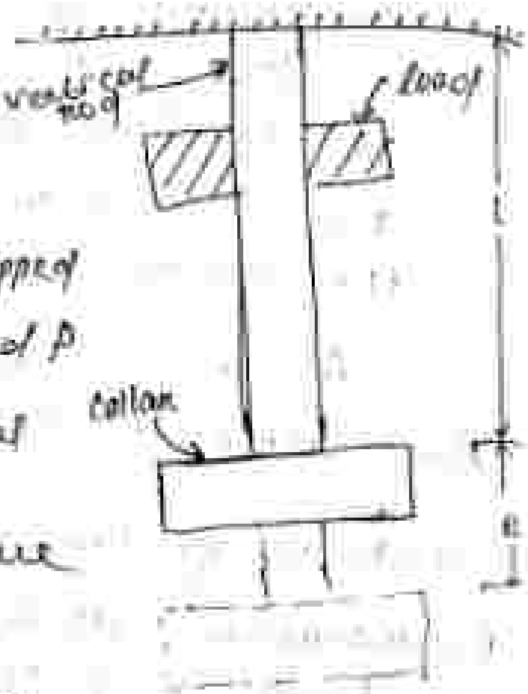
V = Volume of the rod = A×L

h = height through which load is dropped

δL = Extension of the rod due to load P

E = Modulus of elastic of the material of the rod.

σ = stressed induced in the rod due to impact load.



$$\boxed{\text{Stress, } \sigma = \frac{P}{A}\left(1+\sqrt{1+\frac{2AEh}{P.L}}\right)}$$

After knowing the value of 'σ' the strain energy can be obtained.

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* Gradually applied load

When a load is applied in installments i.e. load of 100N to be applied. From a load of 5N then 10N, 15N, 20N - - - - 100N is applied.

⇒ Overally it causes less stress and less strain as compared to suddenly applied load.

⇒ Mostly gradually applied load is found during testing of materials in the SOM Lab.

⇒ Placing of 50 books, one by one on a table and one above the other.

⇒ In this stress strain-curve is triangular.

* Suddenly apply load :-

Total force is applied in one installment. It causes 2 times the stress produced by the gradually applied load.

stress-strain curve is rectangular.

Example:

Weight of 50kg is placed in a weighing balance.

A person slowly sitting on a chair.

Placing a television on table

Placing a bundle of books on table

* Impact Load :

Impact load is moving load. The moving body striking another body creates impact load. It causes many times stress when if the same load is applied gradually.

Ex: Force applied in an accident, Hammering, Lathicharge, striking ball with bat, Falling of an object from the hand and striking the brush.

With the help of ~~the~~ Impact load, the toughness of a material is determined.

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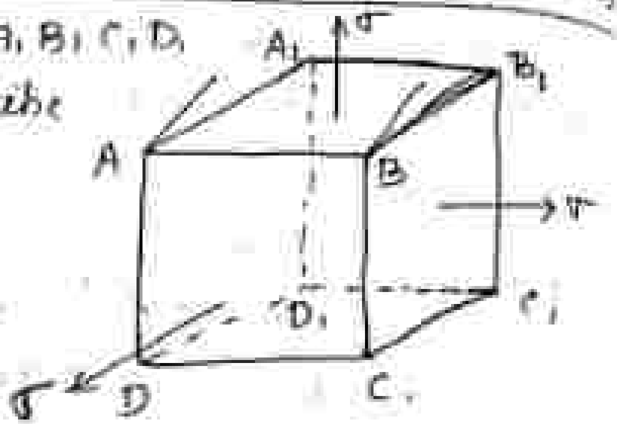
Rel^n b/w Youngs Modulus and Bulk Modulus

Consider a cube $ABCD\,A_1B_1C_1D_1$ as shown in fig. Let the cube be subjected to three mutually ⊥r tensile stresses of equal intensity.



Let σ = stress on the faces

l = Length of the cube, and

E = Young's modulus for the material of the block

Now let us consider the strain (deformation) of one side of the cube (say AB) under the action of three mutually ⊥r stresses. The side will suffer the following three strains.

1. Tensile strain of AB due to stress on the face BB_1CC_1 and AA_1DD_1 which is equal to $\frac{\sigma}{E}$

2. Comp. Strain of AB due to stresses on the faces AA_1BB_1 and DD_1CC_1. This is compressive lateral strain which is equal to $-\mu\frac{\sigma}{E}$

3. Compressive lateral strain of AB due to stress on faces ABCD and $A_1B_1C_1D_1$ which is equal to $-\mu\frac{\sigma}{E}$

Hence total strain (deformation) of AB is given by

$$e_{total} = \frac{dL}{L} = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

$$\boxed{\frac{dL}{L} = \frac{\sigma}{E}(1-2\mu)} \quad \text{—— (i)}$$

Now original volume of the cube, $V = L^3$ —— (ii)

If dL is change in length, then dV is the change in volume.

Differentiating eqn (ii) with respect to L

$$\frac{d}{dL}(V) = \frac{d}{dL}(L^3)$$

$$\Rightarrow \frac{dV}{dL} = 3L^2$$

$$\Rightarrow \boxed{dV = 3L^2 \times dL} \quad \text{—— (iii)}$$

Dividing eqn (iii) by equation (ii).

$$\frac{dV}{V} = \frac{3L^2 \times dL}{L^3} = \frac{3dL}{L}$$

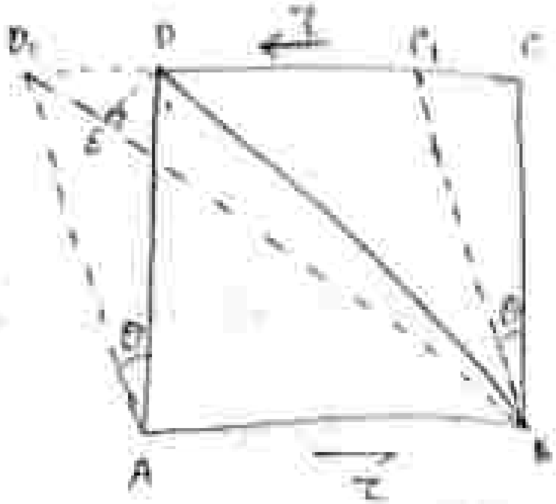
Substituting the value of $\frac{dL}{L}$ from eqn (i), we get

$$\frac{dV}{V} = 3 \times \frac{\sigma}{E}(1-2\mu)$$

$$\Rightarrow \frac{\sigma}{\left(\frac{dV}{V}\right)} = \frac{E}{3(1-2\mu)} \Rightarrow K = \frac{E}{3(1-2\mu)}$$

$$\therefore \boxed{E = 3K(1-2\mu)}$$

Consider a square block ABCD of each side equal to 'a' and subjected to a set of shear stresses of intensity τ on the faces AB, CD and faces AD and CB. Let the thickness of the block normal to the plane of the paper is unity.



Due to the set of shear stress the diagonal BD will experience a tensile stress of magnitude q whereas the diagonal ~~BD~~ AC will experience a comp. stress of magnitude q.

- Due to these stresses the diagonal BD will be elongated whereas the diagonal AC will be shorted. Let us consider the joint effect of these two stresses on the diagonal BD.

- Due to the tensile shear stresses q along diagonal BD, there will be a tensile strain in diagonal BD. Due to the comp. shear stress q along the diagonal AC, there will be a tensile strain in the diagonal BD due to lateral strain.

Let μ = Poisson's ratio for the material of the b
E = Youngs modulus

Now tensile strain in diagonal BD due to tensile shear stress q along BD

$$= \frac{\text{Tensile stress along BD}}{E} = \frac{q}{E}$$

Tensile strain in BD due to comp. stress τ along AC

$$= \mu \times \frac{\tau}{E}$$

$\therefore$ Total tensile strain along diagonal BD

$$= \frac{\tau}{E} + \frac{\mu \times \tau}{E} = \frac{\tau}{E}(1+\mu)$$

Similarly total comp. strain in diagonal AC

$$= \frac{\tau}{E}(1+\mu)$$

We know that total tensile strain in the diagonal BD is equal to half the shear strain.

i.e Total tensile strain in diagonal BD

$$= \frac{1}{2} \times \text{shear strain} \qquad \left(\because G = \frac{\tau}{\phi} \right)$$

$$= \frac{1}{2} \times \frac{\text{shear stress}}{G}$$

$$= \frac{1}{2} \times \frac{\tau}{G}$$

Equating the ~~two~~ tensile strain along diagonal BD, we get

$$\frac{\tau}{E}(1+\mu) = \frac{1}{2} \times \frac{\tau}{G} = \frac{\tau}{2G}$$

$$\Rightarrow \boxed{E = 2G(1+\mu)}$$

or
$$\boxed{G = \frac{E}{2(1+\mu)}}$$

Relation b/w E, G & K

We know that $E = 2G(1+\mu)$ — (i)

And $E = 3K(1-2\mu)$ — (ii)

from eq^n (i) $1+\mu = \frac{E}{2G}$

$$\therefore \mu = \frac{E}{2G} - 1$$

Putting the value in eq^n (ii)

$$E = 3K\left[1 - 2\left(\frac{E}{2G} - 1\right)\right] = 3K\left[1 - \frac{E}{G} + 2\right]$$

$$E = 3K\left(3 - \frac{E}{G}\right) = 3K\left[\frac{3G-E}{G}\right]$$

$$EG = 3K(3G - E) = 9KG - 3KE$$

$$\therefore EG + 3KE = 9KG$$

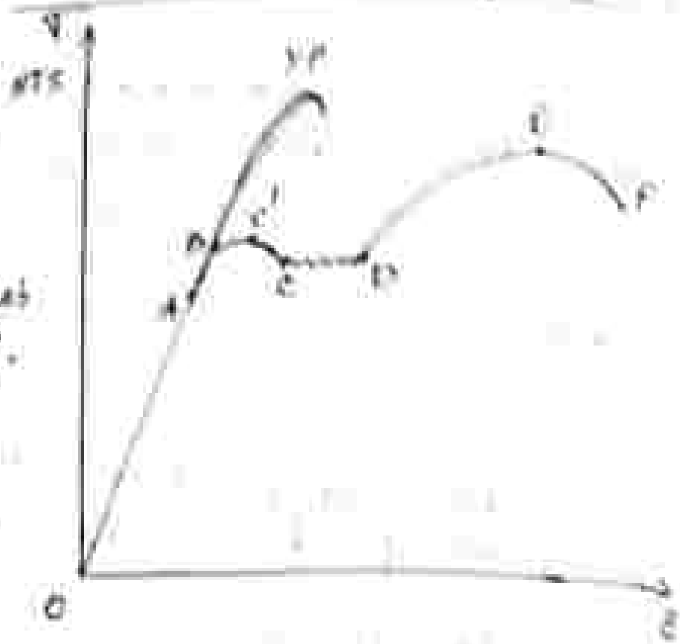
$$\therefore E(3K+G) = 9KG$$

$$\therefore \boxed{E = \frac{9KG}{3K+G}}$$

Ans

Tensile Test of a Mild Steel (Stress Strain Diagram)

(Mild steel Bar)



A → Proportional limit

The curve OA is linear & Hookes law is valid upto this limit only.

B → Elastic limit

After unloading, entire strain will be released.

C' → Upper yield point

C → Lower yield point (Actual yield point)

When the specimen is stressed beyond the elastic limit, the strain increases more quickly than the stress. This happens because a sudden elongation of the specimen takes place, without an appreciable increase in the stress. This phenomenon is called yielding. The stress corresponding to the pt. B is called yield stress.

The fall of stresses from C' to C is due to slipping of carbon atoms in the molecular str. of mild steel. For mild steel A, B, C are very close to each other.

→ After A.B the material shows plastic behaviour.

C-D → CD is called yield point stress. The stress corresponding to yield stress is called actual yield point. Because mild steel behave as plastic from this point onwards.

→ At point D the specimen regains some strength & higher value of stresses are required, for higher strains from D to E is the region of strain hardening. During strain hardening the material undergoes the changes in crystalline structure resulting in increase resistance of the material to further deformation.

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Chapter 8.0 THIN CYLINDER AND SPHERICAL SHELL UNDER INTERNAL PRESSURE

Introduction: Thin cylinders and vessels actually used to store fluids (gas, liquid etc.) under pressure such as tanks, boilers, compressed air receiver etc. Generally walls of such vessels are very thin as compared to their diameters. These vessels when empty, are subjected to atmospheric pressure internally as well as externally. In such a case resultant pressure on the walls of the shell is zero. But whenever a vessel is subjected to internal pressure (due to steam, compressed air etc.) its walls are subjected to tensile stresses.

If the thickness of the wall of a shell is less than 1/10 th to 1/15 th of its diameter, it is known as thin shell.

$$\boxed{t \le \left(\frac{1}{10}d \text{ to } \frac{1}{15}d\right)}$$

Failure of a Thin cylindrical shell due to I.P

Internal pressure $\longrightarrow$ tensile stress

If stress exceeds permissible limit, the cylinder is likely to fail in any one of the following two ways:

1. It may split up into two troughs and
2. " " " " two cylinders.

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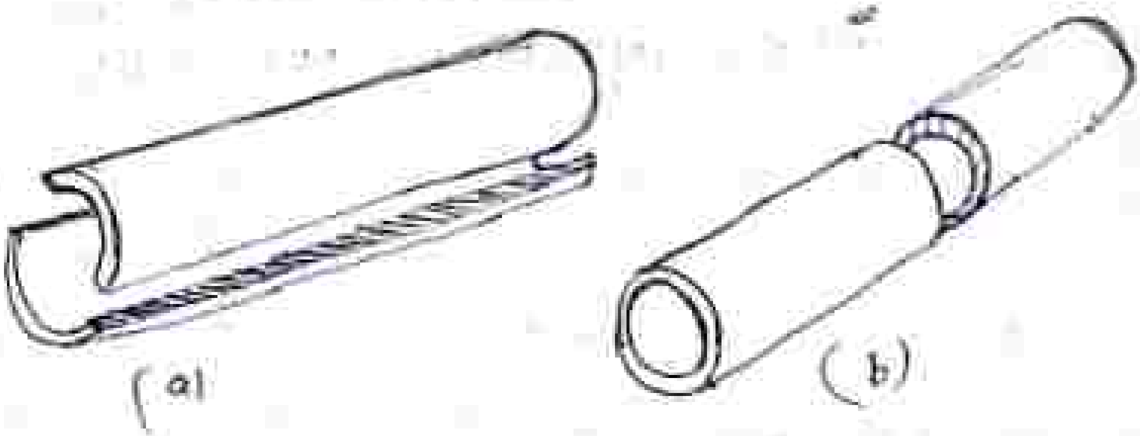


(a) (b)

Stresses in a Thin Cylindrical Shell

subjected to two types of stress.

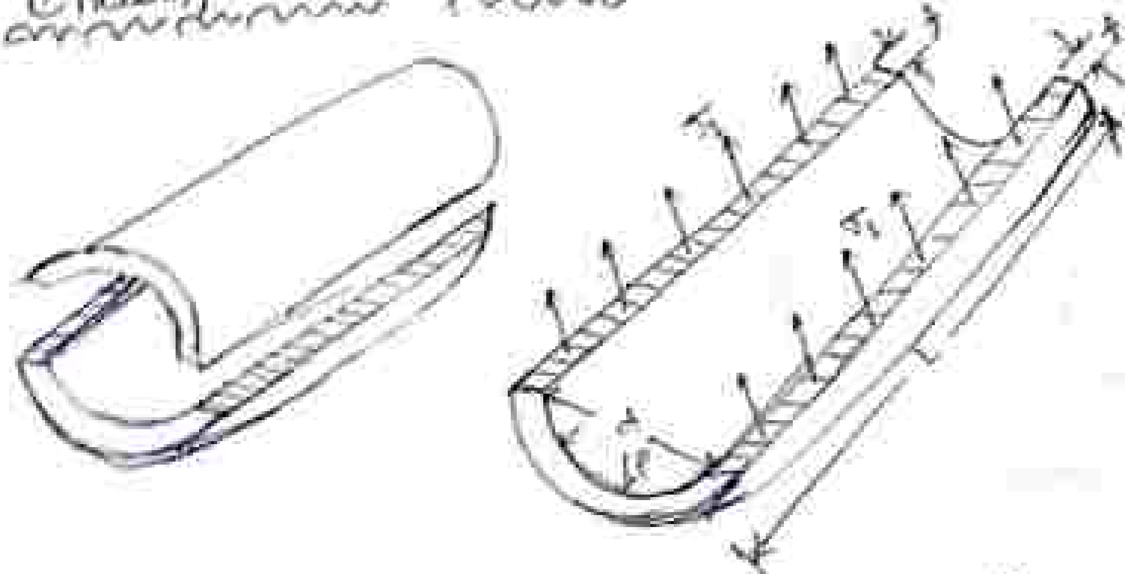
1. Circumferential stress or (hoop stress)
2. Longitudinal stress

Note:- In case of thin shells the stresses are assumed to be uniformly distributed throughout the wall thickness. However in the case of thick cylindrical shell, the stresses are no longer uniformly distributed, hence problem becomes complex.

N.B. The above theory also holds good, when shell is subjected to compressive stress

(Thin cylindrical subjected to IP)

Circumferential stress



Circumferential stress will be set up if bursting of the cylinder takes in the above way.

Let p = Internal fluid pressure

d = " dia of cylinder

t = thickness of the wall of the cyl.

σ_1 = Circumferential / hoop stress of the material

If Force due to fluid press > circumferential st. then bursting will takes place.

In the limiting case, the two forces should be equal.

Force due to fluid pressure = $p \times$ Area on which p is acting

$= p \times L \times d$ ——(i)

Force due to circumferential stress

= $\sigma_1 \times$ Area on which σ_1 acts

$= \sigma_1 \times (L \times t + L \times t)$

$= \sigma_1 \times 2Lt = 2\sigma_1 Lt$ ——(ii)

Equating (i) and (ii), we get

$$p \times d \times l = 2\sigma_1 \times t \times l$$

$$\therefore 2\sigma_1 \times t = p \times d$$

$$\therefore \boxed{\sigma_1 = \frac{p \times d}{2t}}$$ (Hoop stress)

Another way.

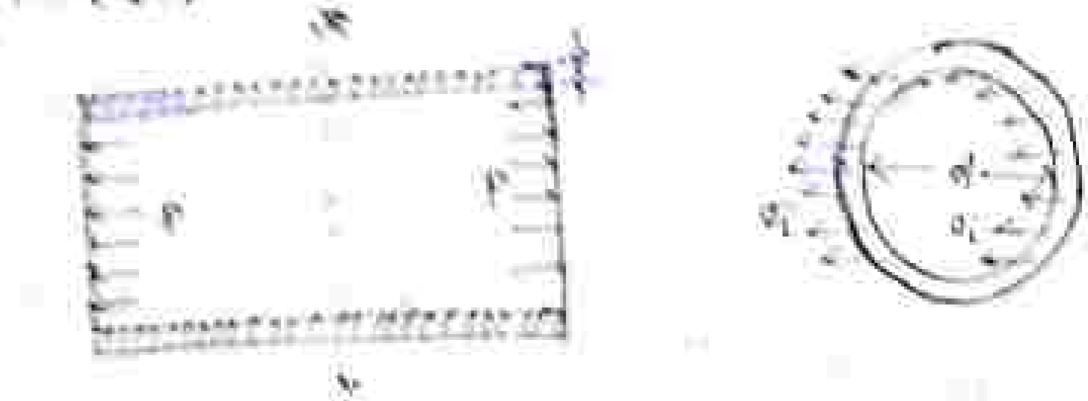
$$\text{Hoop stress}, \ \sigma_1 = \frac{\text{Total pressure}}{\text{Resisting section}}$$

$$= \frac{p\,d\,l}{2\,t\,l} = \frac{pd}{2t} \ (\sigma_1 \text{ of hoop } \ldots)$$

Note: If η is the efficiency of the riveted joint of the shell, then

$$\boxed{\text{stress } (\sigma_1) = \frac{pd}{2t\eta}}$$

Longitudinal Stress:



Force due to fluid pressure = p × area on which p is act

$$= p \times \frac{\pi}{4} d^2$$

Resisting force = σ_2 × area on which σ_2 is acting

$$= \sigma_2 \times \pi d \times t$$

$p \times \frac{\pi}{4} d^2 = \sigma_a \times \pi d \times t$

$$\Rightarrow \sigma_L = \frac{pd}{4t} \quad \text{(tensile)}$$

<u>Note</u> (i) Longitudinal stress is half of the σ_c.

$$\boxed{\sigma_L = \frac{pd}{4t\eta}}$$

(ii) If the thickness of the thin cylinder is to be determined, then $\left(\sigma_c = \frac{pd}{2t}\right)$ is used.

(iii) If max. permissible stress in the material is given, stress should be taken σ_c.

(iv) While using eqn for σ_c & σ_L, units should be same.

<u>Effect of Internal pressure on the dimensions of a thin cylindrical shell</u> :-

When a fluid having internal pressure p is stored in a thin cylindrical shell, due to internal pressure of the fluid the stresses set up at any point of the material of the shell are

(i) Hoop/ circumferential (σ_h) acting on longitudinal ~~plane~~ section.

(ii) Longitudinal stress (σ_L) acting on the circumferential sec.

These stresses are principal stresses, acting on principal planes. The stresses in the third plane is zero as thickness (t) of the cylinder is very small. Actually the stresses in the third principal plane is radial stress which is very small for thin cylinder and can be neglected.

Let p = internal pressure of fluid

l = Length

d = Diameter

t = Thickness

E = Modulus of Elasticity

σ_h = Hoop stress

σ_L = Longitudinal stress

μ = Poissons ratio

δd = change in diameter due to stress set up in the material

δl = change in length

δV = change in volume.

We know that $\sigma_h = \frac{pd}{2t}$

$$\sigma_L = \frac{pd}{4t}$$

Let e_h = Circumferential strain

e_L = Longitudinal strain

Circumferential strain :-

$$e_h = \frac{\sigma_h}{E} - \mu \frac{\sigma_L}{E}$$

$$= \frac{pd}{2tE} - \mu \frac{pd}{4tE}$$

$$\boxed{\therefore e_h = \frac{pd}{2tE}\left[1 - \frac{\mu}{2}\right]}$$

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Longitudinal strain :-

$$e_L = \frac{\sigma_L}{E} - \mu \frac{\sigma_h}{E}$$

Circumference = πd

$$= \frac{pd}{4tE} - \mu \frac{pd}{2tE}$$

$$\boxed{\therefore e_L = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)}$$

change in diameter :-

$$\boxed{\delta d = \frac{pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)}$$

Change in length :-

$$\boxed{\delta L = \frac{pdL}{2tE}\left(\frac{1}{2} - \mu\right)}$$

Volumetric strain :-

$$\boxed{e_v = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)}$$

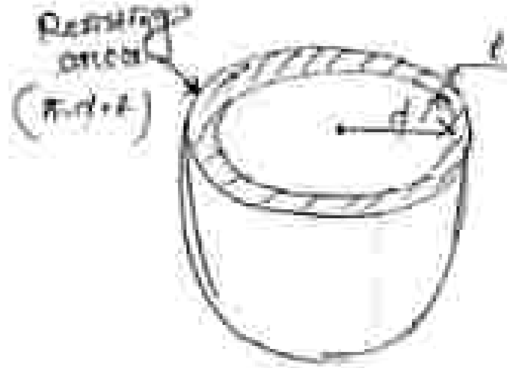
Also change in volume $(\delta V) = V\left(2e_h + e_L\right)$

Thin spherical shells.

Force (P) = $p \times \frac{\pi}{4} d^2$

The area resisting this force = $\pi . d . t$

∴ Hoop or circumferential stress (σ_h) induced in the material of the shell is given by



$$\sigma_h = \frac{\text{Force } P}{\text{Area resisting the force } P}$$

$$= \frac{p \times \frac{\pi}{4} d^2}{\pi d t} = \frac{pd}{4t}$$

where σ_h is tensile in nature.

The fluid inside the shell is also ha tendency to split the shell into two hemispheres along y-y axis.

Then it can be shown that the tensile hoop stress will also be equal to $\frac{pd}{4t}$. Let this stress be σ_L or σ_2, $\sigma_2 = \frac{pd}{4t}$

This stress will be right angle to σ_h.

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Change in dimensions of a thin spherical shell due to an internal pressure :-

Max. shear stress $= \frac{\sigma_1 - \sigma_2}{2} = \frac{pd}{4t} - \frac{pd}{4t} = 0$

$\therefore$ Strain in any one direction is given by

$e = \frac{\sigma_h}{E} - \mu \frac{\sigma_L}{E}$ $\left(\sigma_h = \sigma_L = \frac{pd}{4t}\right)$

$= \frac{\sigma_h}{E}(1-\mu)$

Strain in any direction $= \frac{\delta d}{d}$

$= \frac{pd}{4tE}(1-\mu)$

Volumetric strain :-

Let V = original volume

$= \frac{4}{3}\pi r^3 = \frac{\pi}{6} d^3$

~~original~~ Final volume due to pressure,

$V + \delta V = \frac{\pi}{6}(d + \delta d)^3$

$\therefore$ Volumetric strain,

$\frac{\delta V}{V} = \frac{(V + \delta V) - V}{V} = \frac{\frac{\pi}{6}(d+\delta d)^3 - \frac{\pi}{6} \times d^3}{\frac{\pi}{6} d^3}$

$= \frac{d^3 + 3d^2 \cdot \delta d - d^3}{d^3}$ (neglecting higher power of δd.)

$= \frac{3\delta d}{d}$, i.e. $3 \times \frac{pd}{4tE}(1-\mu)$

$$\boxed{\therefore \text{ volumetric strain } (e_v) = 3 \times \frac{pd}{4tE}(1-\mu)}$$

$$\text{and } \delta V = V \cdot 3e = \frac{\pi}{6} d^3 \times 3 \times \frac{pd}{4tE}(1-\mu)$$

$$\therefore \delta V = \frac{\pi p d^4}{8tE}(1-\mu)$$

CHAPTER-3.0

TWO DIMENSIONAL STRESS SYSTEMS

Introduction:

In the previous chapter we have discussed the direct tensile, compressive stress as well as shear stresses. These stresses ~~were~~ acting in a plane, which was at right angle to the line of action of the force. In many engineering problems both direct (tensile or compressive) and shear stresses are acting at the same time. In such situation the resultant stress across any section will be neither normal nor tangential to the plane. In this chapter the stresses, acting on an inclined plane (or oblique section) will be analysed.

Principal Plane:

In a 3-D body, there may be three planes mutually ⊥r to each other which carry direct stresses only, and no shear stress. A little consideration will show that out of these three direct stress one will be max, the other will be minimum, and the third an intermediate between the two. These particular planes, which have no shear stress, are known as principal planes.

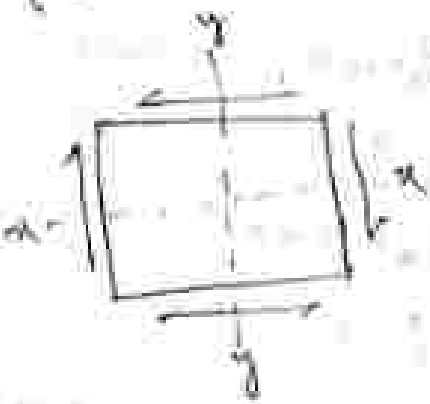
Principal Stress :-

The magnitude of direct stress, across a principal plane, is known as principal stress. The determination of principal planes, and principal stress is an important factor in the design of various structures & machine components.

Methods for the stresses on an oblique section of a Body

1. Analytical method
2. Graphical method

Sign Conventions for Analytical Method :-

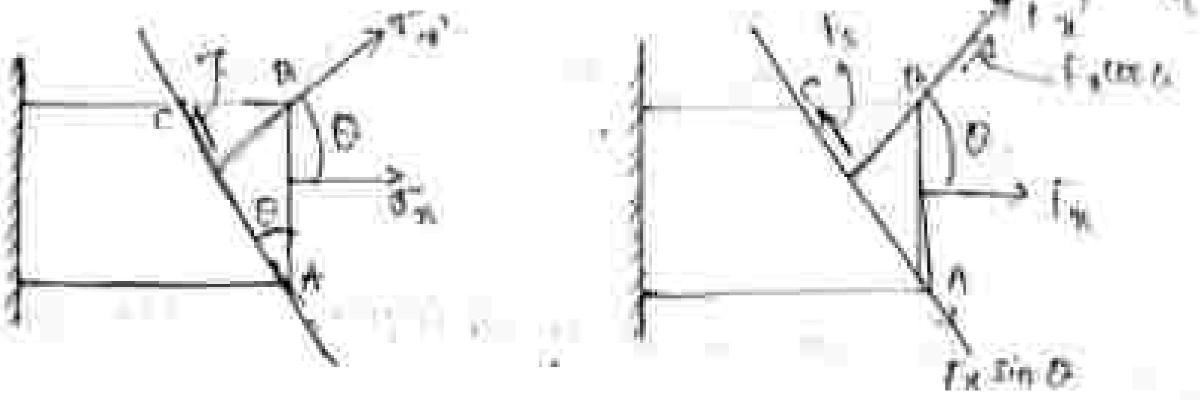


1. All the tensile stresses are taken as positive and all the comp. stresses are taken as -ve.
2. If shear stress will tends to rotate the elements in clockwise direction then it is +ve otherwise -ve.

Shear stress on vertical face $(x-x)$ +ve.
Horizontal " $(y-y)$: -ve.

Transformation of stress on oblique section (Analytic...

1. A member subjected to a Direct stress in one plane



Let σ_x = Tensile stress across the face AB (x-x axis)

θ = Angle which the oblique section AC makes with AB or with the x-axis.

$\sigma_{x'}$ or σ_n = Normal stress ~~across~~ to the section AC

F_s : τ = Tangential stress to the surface AC. $\left(\tau = \frac{F}{A}\right)$

F_x = Force across the face AB

$F_x' = F_n$ = normal force to the face AC

Normal stress across the section AC

$$F_x' = F_x \cos\theta$$

$$F_s = -F_x \sin\theta$$

$$F_x = \sigma_x \times (AB \times t)$$

$$F_x' = \sigma_{x'} \times (AC \times t)$$

$$F_s = \tau \times (AC \times t)$$

$$F_x' = F_x \cos\theta$$

$$\sigma_{x'}(AC \times t) = \sigma_x \times (AB \times t) \cos\theta$$

$$\Rightarrow \sigma_{x'} = \sigma_x \times \frac{AB}{AC} \cos\theta$$

$$\Rightarrow \sigma_{x'} = \sigma_x \times \cos\theta \times \cos\theta$$

$$\boxed{\sigma_{x'} = \sigma_n = \sigma_x \cos^2\theta} \quad \text{---(1)}$$

$(\sigma = \sigma_x)$

or $\boxed{\sigma_n = \sigma \cos^2\theta}$

$$F_s = \tau \times (AC \times l)$$

$$-F_x \sin\theta = \tau \times (AC \times t)$$

$\Rightarrow$ σ ⟵ 1

$$\Rightarrow -\sigma_x \times (AB \times t) \sin\theta = \tau \times AC \times t$$

$$\Rightarrow \tau = \sigma_x \times \left(\frac{AB}{AC}\right) \sin\theta$$

$$\Rightarrow \tau = -\sigma_x \cos\theta \cdot \sin\theta$$

or $\boxed{\tau = \frac{-\sigma_x \sin 2\theta}{2}}$ ⟵ (ii)

From the equation (i), it will be seen that, normal stress will be max. when $\cos\theta$ is max. or $\cos^2\theta$ is max.

max. value of $\cos\theta = 1$, $\cos 0° = 1$.

But when $\theta = 0°$, the section AC will coincide with section AB. But the section AB is normal to the line of axial loading. This means plane normal to the axis of loading will carry the max. normal stress.

$\boxed{\therefore \text{ Maximum normal stress} = \sigma \cos^2\theta = \sigma \cos^2 0° = \sigma}$

From the equation (2), tangential (shear stress) across the section AC will be maximum when $\sin 2\theta$ is maximum.

$\sin 2\theta$ is max. when $\sin 2\theta = 1$ or $2\theta = 90^\circ$ or 270°

or $\theta = 45^\circ$ or 135°

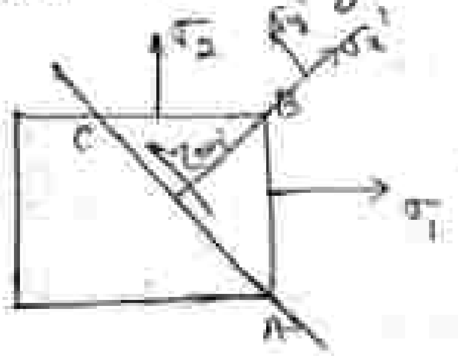
$\therefore$ Shear stress will be maximum on two planes inclined at 45° and 135° to the normal section as

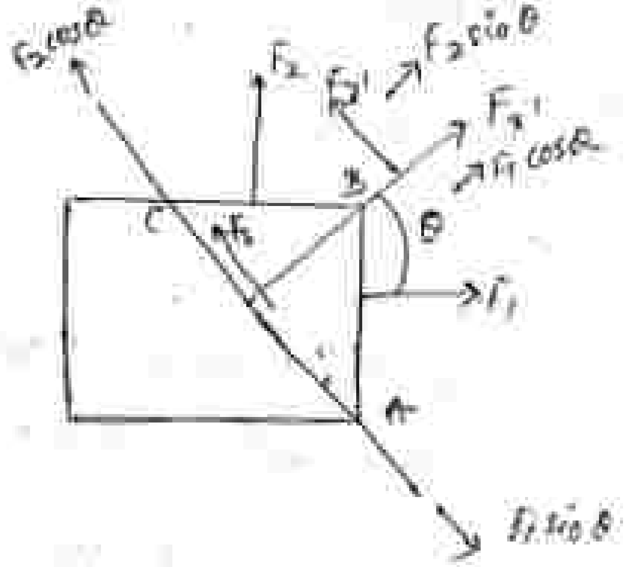
$\therefore$ Max. value of shear stress $= \frac{\sigma}{2} \sin 2\theta$

$= \frac{\sigma}{2} \sin 90^\circ = \frac{\sigma}{2}$

Hence max. value of normal stress is double to that of max. value of shear stress.

2. Stresses on an Oblique Section of a Body subjected to Direct stress or principal stress on two mutually ⊥r planes.



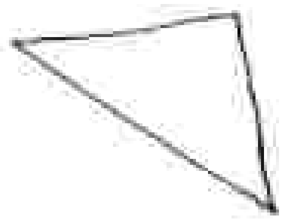


$F_1 = \sigma_1 (AB \times t)$

$F_2 = \sigma_2 (BC \times t)$

$F_n' = \sigma_n' (AC \times t)$

$F_t' = \tau_{ut} (AC \times t)$



$F_{x'} = F_1 \cos\theta + F_2 \sin\theta$

$\sigma_{x'} (AC \times l) = \sigma_1 (AB \times l) \cos\theta + \sigma_2 (BC \times l) \sin\theta$

$\sigma_{x'} = \sigma_1 \times \frac{AB}{AC} \cos\theta + \sigma_2 \frac{BC}{AC} \sin\theta$

$\sigma_{x'} = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta$ $\quad [1+\cos 2\theta = 2\cos^2\theta]$ $\quad [1-\cos 2\theta = 2\sin^2\theta]$

$= \sigma_1 \left(\frac{1+\cos 2\theta}{2}\right) + \sigma_2 \left(\frac{1-\cos 2\theta}{2}\right)$

$= \frac{\sigma_1 + \sigma_1 \cos 2\theta}{2} + \frac{\sigma_2 - \sigma_2 \cos 2\theta}{2}$

$= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$

$\therefore \boxed{\sigma_{x'} = \sigma_n = \text{normal stress} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta}$

For $\sigma_{y'}$, $\theta = 90 + \theta$

$\sigma_{y'} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2(90+\theta)$

$\boxed{\sigma_{y'} = \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta}$

σ_1 & σ_2 are two principal stress and also principal stress.

Tangential stress (or shear stress) along section AC

$F_{s'} = F_2 \cos\theta - F_1 \sin\theta$

$\tau_{xy'} (AC \times t) = \sigma_2 (BC \times t) \cos\theta - \sigma_1 (AB \times t) \sin\theta$

$\tau_{xy'} = \sigma_2 \frac{BC}{AC} \cos\theta - \sigma_1 \frac{AB}{AC} \sin\theta$

$\tau_{xy'} = \sigma_2 \sin\theta \cdot \cos\theta - \sigma_1 \sin\theta \cdot \cos\theta$

$= \frac{\sigma_2 \sin 2\theta}{2} - \frac{\sigma_1 \sin 2\theta}{2}$

or $\boxed{\tau_{xy'} = \frac{-(\sigma_1 - \sigma_2) \sin 2\theta}{2}}$

In many books, $\boxed{\tau_{xy'} = \frac{(\sigma_1 - \sigma_2) \sin 2\theta}{2}}$

At $\theta = 45^\circ$ or 135°, $\tau_{xy'}$ will be maximum,

or, $\boxed{|\tau_{max}| = \left|\frac{\sigma_1 - \sigma_2}{2}\right|}$

At this plane both normal stresses and $\sigma_{x'}$ and $\sigma_{y'}$ will be same of

$\boxed{\sigma_{x'} = \sigma_{y'} = \frac{\sigma_1 + \sigma_2}{2}}$

The resultant stress on the section AC will be

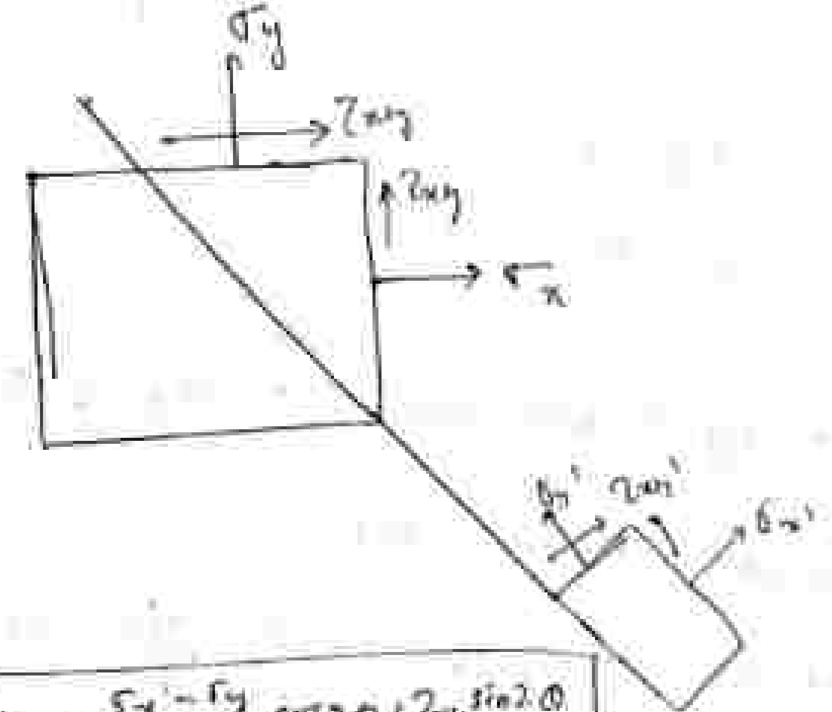
$$R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

or $R = \sqrt{\sigma_{x'} + \tau_{xy'}}$

Inclination of resultant ~~plane~~ stress with the normal of the inclined plane is given by.

$$\boxed{\tan\theta = \frac{\sigma_t}{\sigma_n}}$$

3. A member subjected to direct stresses in two mutually ⊥r plane accompanied by a simple shear stress.



$$\boxed{\sigma_{x'} = \sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta}$$

$$\boxed{\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta}$$

$$\boxed{\tau_{xy'} = \sigma_t = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta}$$

Note
The summation of normal stresses on all the plane remains constant.

$$\boxed{\sigma_x + \sigma_y = \sigma_1 + \sigma_2 = \sigma_{x'} + \sigma_{y'} = \text{constant}}$$

For principal planes $\tau_{xy'}$ will be equal to zero.

or $\frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta = 0$

$$\boxed{\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

$\theta_{p_1} = \theta_p$ &

$\theta_{p_2} = 90 + \theta_p$ ... .

Therefore by substituting $\theta = \theta_p$ & $\theta_p + 90°$,

the principal stresses are,

Major principal stress $(\sigma_1) = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

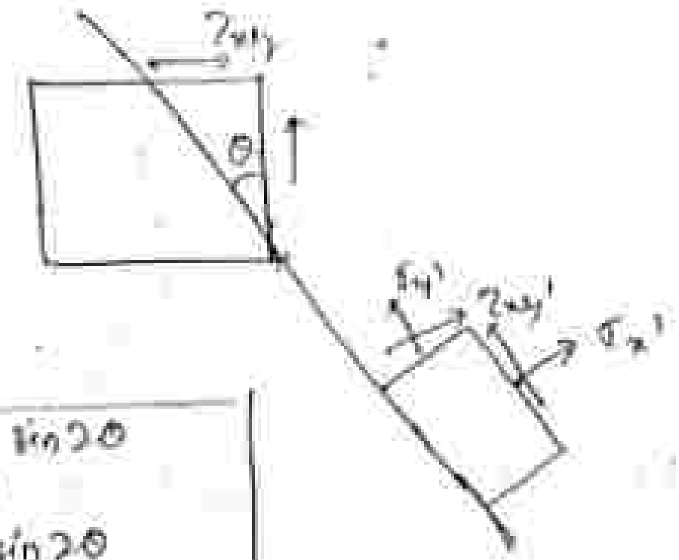
Minor principal stress $(\sigma_2) = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

For maximum shear stress, $\frac{d(\tau_{xy'})}{d\theta} = 0$.

at $\theta = \theta_p + 45°$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}$$

4. A member subjected to a simple shear stress (Case of pure shear)



Normal stress.

$$\boxed{\begin{aligned} \sigma_x' &= \sigma_n = \tau_{xy}\sin 2\theta \\ \sigma_y' &= -\tau_{xy}\sin 2\theta \\ \tau_{xy}' &= -\tau_{xy}\cos 2\theta \end{aligned}}$$

Maximum and minimum normal stress (principal planes)

shear stress should be zero.

$$-\tau_{xy}\cos 2\theta = 0$$

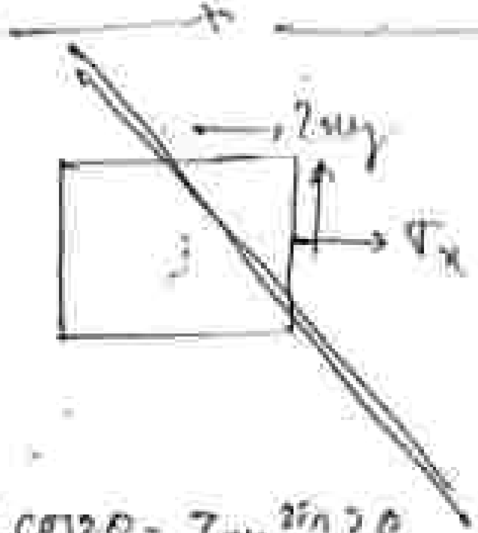
if $2\theta = 90$ or 270°

or $\theta = \cancel{0}\ 45^\circ$ or 135°

$$\therefore \boxed{\begin{aligned} \sigma_1 &= \tau_{xy} \\ \sigma_2 &= -\tau_{xy} \end{aligned}}$$

* σ_1 & σ_2 are both direct stress & principal stresses.

5. Stresses on an Oblique Section of a Body subjected to a Direct Stress in one plane and Accompanied by a Single Shear Stress



$$\sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau = \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

Angle of inclination, $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x}$

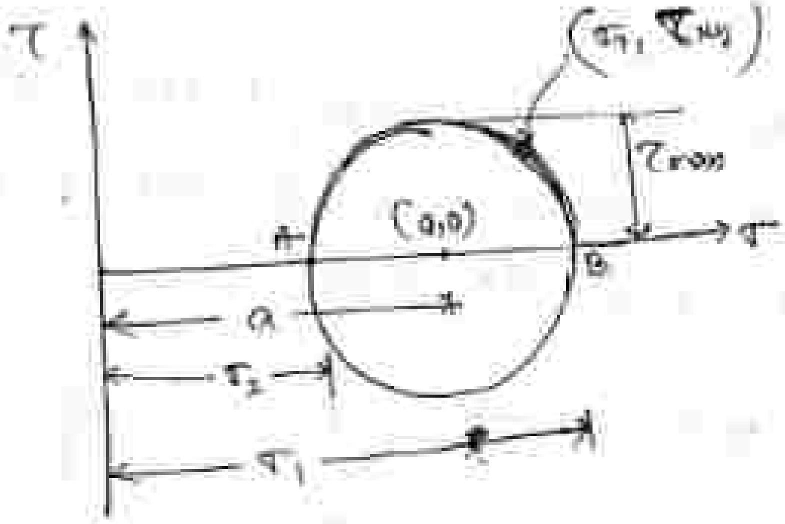
Maximum principal stress,

$$\sigma_{1} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{2} = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

Graphical Method or Mohr's stress circle method

Mohr's circle is the locus of the position of the normal and shear stress magnitude acting on an element at various place.



$$\text{Centre} = (a, 0) = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$$

where, $a = \frac{\sigma_x + \sigma_y}{2}$

$$\text{Radius}, \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Properties of Mohr's stress Circle

①. Mohr's circle is always symmetrical about normal stress axis. It cuts the normal stress axis at two points A & B, the coordinate of which represents major principal stress and minor principal stress respectively.

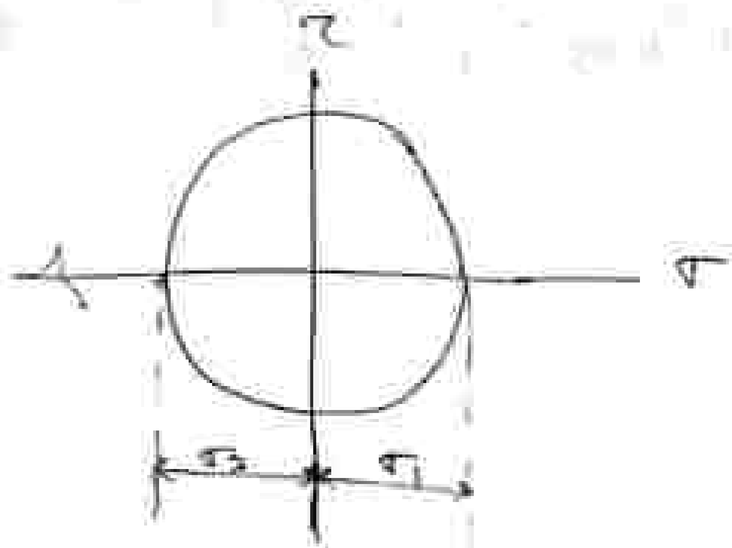
② For the case of pure shear, Mohr's circle will be symmetrical about both the axis & its centre will coincide with the origin.

For pure shear, $\sigma_1 = +\tau_{xy}$

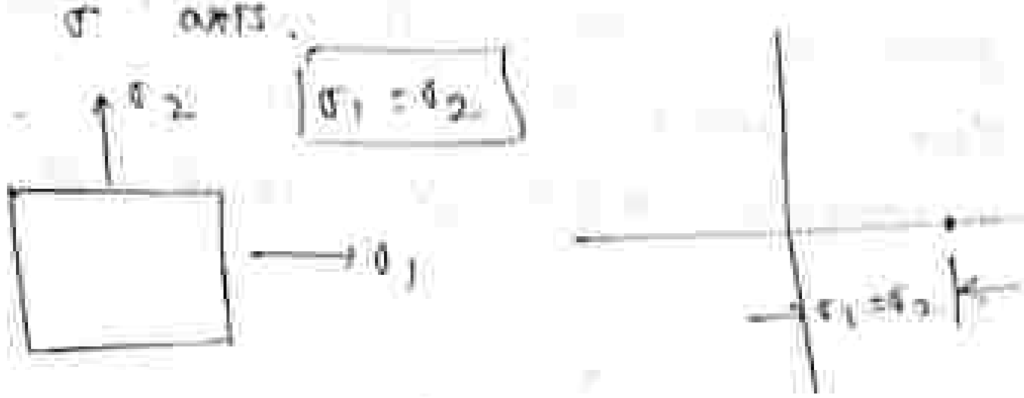
$\sigma_2 = -\tau_{xy}$

$$\therefore \quad a = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2} = 0$$

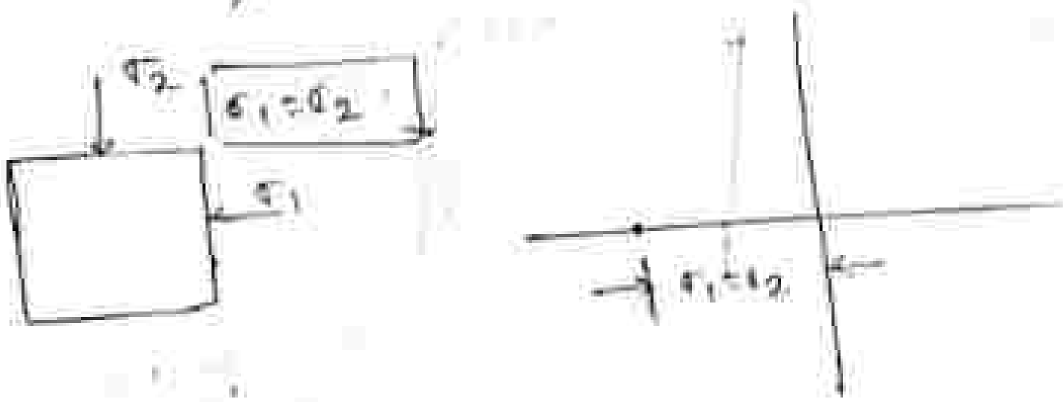
Centre $= (a, 0) = (0, 0)$



③. If principal stresses on two mutually ⊥ planes are equal and of same sign, then Mohr's circle for such case will become a point which will lie on σ axis.



$$\text{Radius} = \tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 0$$

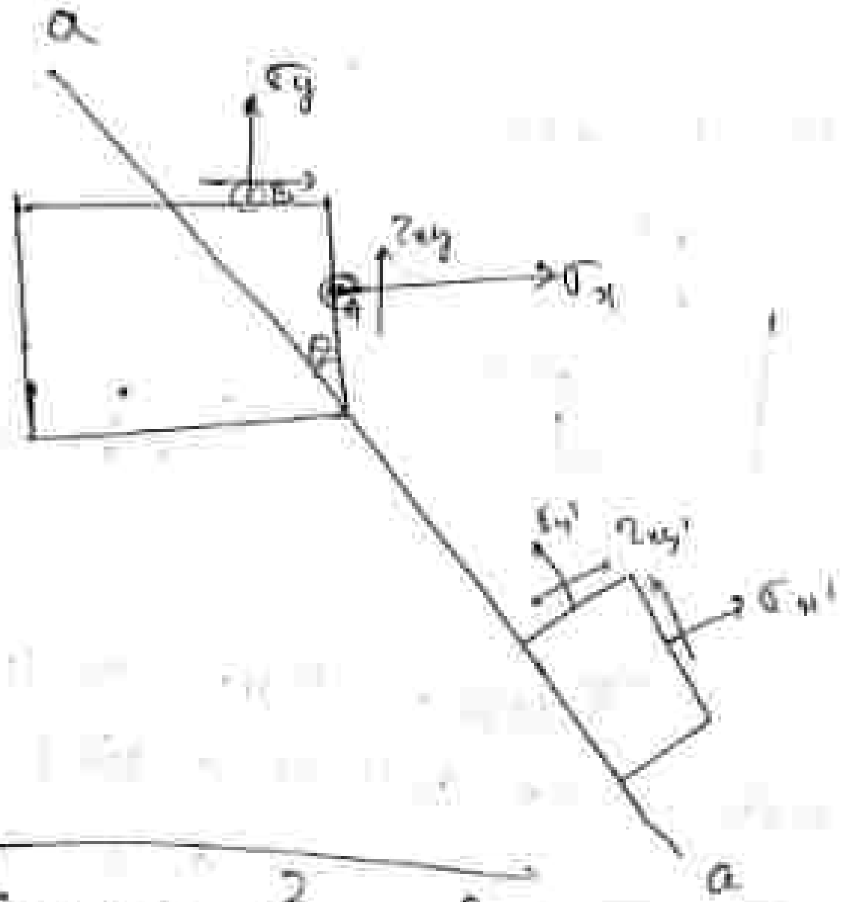


Therefore Mohr's circle for an element on the bottom tank will be a point which will lie on $-ve$ σ axis. In such a case there will be infinite number of principal planes.

Construction of Mohr's Circle.

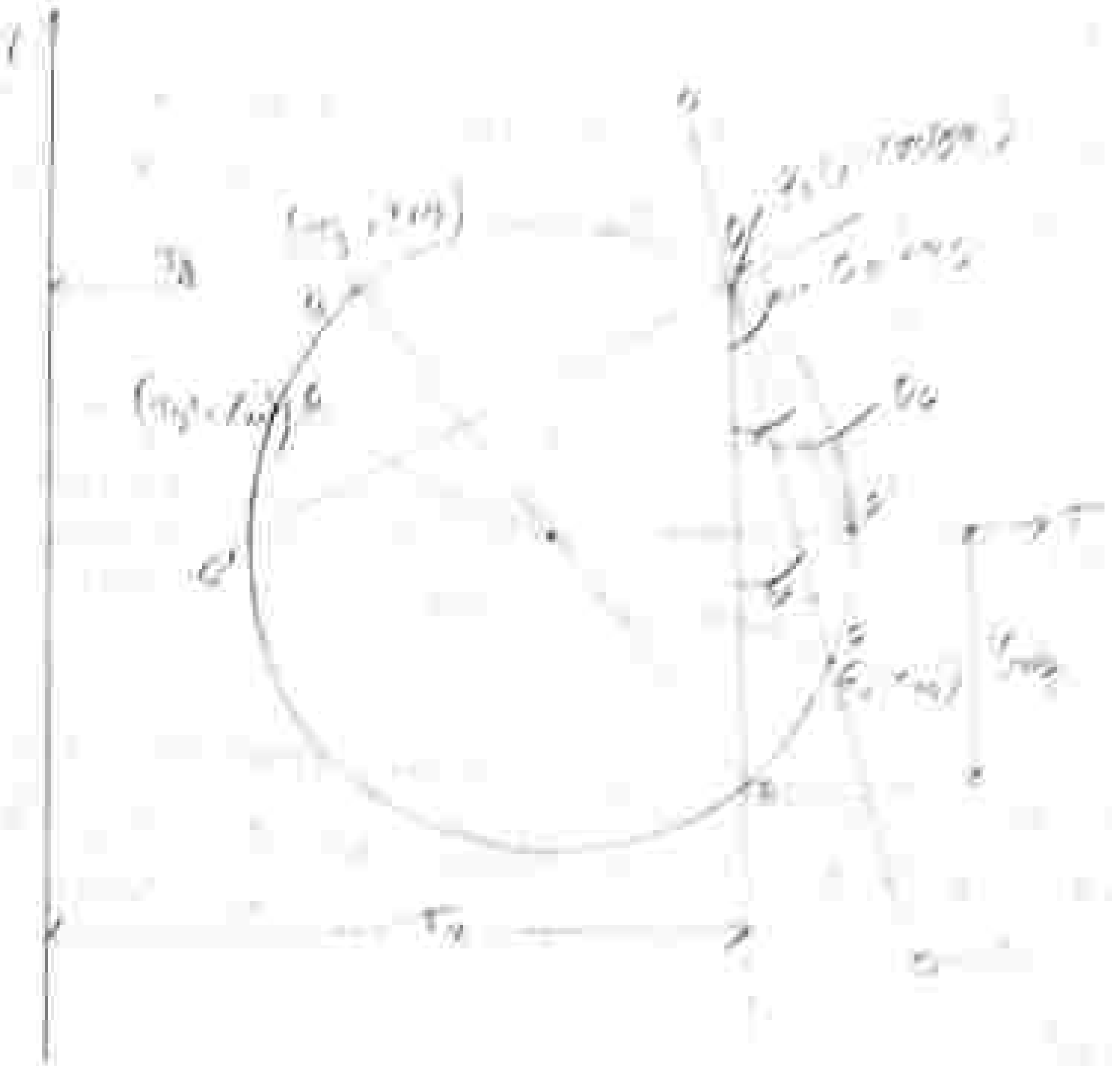
Sign convention of shear stress for Mohr's circle:

If shear stress produces clockwise couple about the centre of the element then it will be plotted above the σ axis (+ve) & if it produces anticlockwise couple then it will be plotted below σ axis.



$$R = \tau_{max} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\text{Centre}, \quad C = \frac{\sigma_x + \sigma_y}{2},$$



- After drawing the circle mark the point A and B on the circle which represents the coordinates of stresses on vertical plane A and horizontal plane B.
- Since on vertical plane A, shear produces anticlockwise couple and normal stress is tensile on the circle, point A lies below σ axis.
- Similarly on horizontal plane B, shear produces clockwise couple. Therefore it lies above σ axis. Note that if A & B on planes normal to each other.

then these points are diametrically opposite.

- From point A draw a vertical plane which intersects the circle at 'O' which is called pole or origin of the plane.

- To find horizontal plane draw a line from 'O' joining B. Therefore OA represents vertical plane whereas OB represents horizontal plane.

- To find normal and shear stress on any plane (a-a) which is inclined at $\theta°$ from the vertical, draw a line from pole at angle θ with vertical.

- The intersection of this line on the circle at P represents the co-ordinate of 'σ_x' and 'τ_{xy}'.

- Similarly point Q shows the co-ordinate of σ_y and 'τ_{xy}' which is diametrically opposite to point P.

- To find principal planes with the vertical, join P' & Q' with origin O. Therefore OP will give the angle of major principal plane.

SFD / BMD

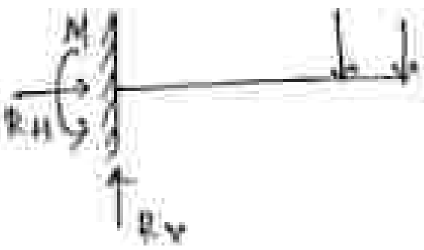
A structural member which is acted upon by a system of external loads at right angle to its axis is known as beam.

→ Whenever a horizontal beam is loaded with vertical loads, sometimes, it bends (i.e. deflects) due to the action of the loads.

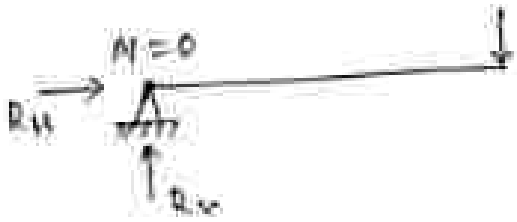
→ The amount with which a beam bends, depends upon the amount and types of loads, length of the beam, Elasticity of the beam and type of the beam.

Types of Support :-

① Fixed support



② Hinge support

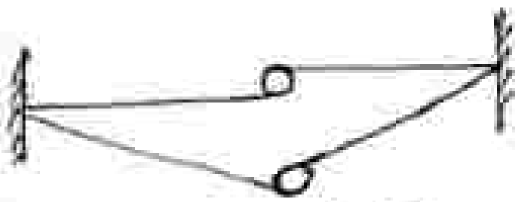


③ Roller support

④ Internal Hinge

$M = 0,\ R_H = 0,\ R_V = 0,\ F_N = 0$

⑤

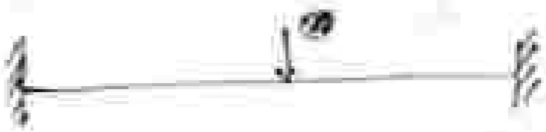


$F_N = 0 \quad M = 0$

Types of Beam :-

① Simply supported Beam

② Fixed Beam



③ Cantilever Beam

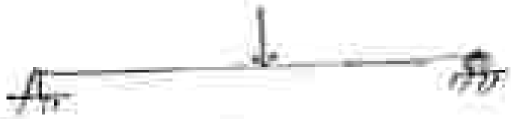
④ Overhanging Beam

⑤ Prop cantilever beam

⑥ Continuous beam

Types of Loadings

1) Concentrated or point load



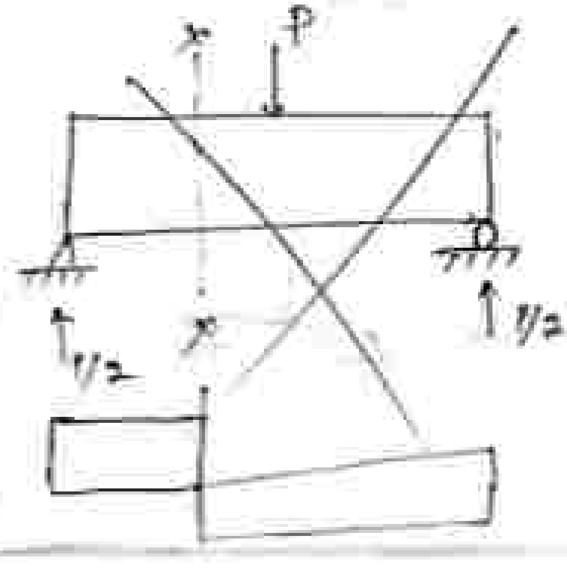
2) Uniformly distributed load

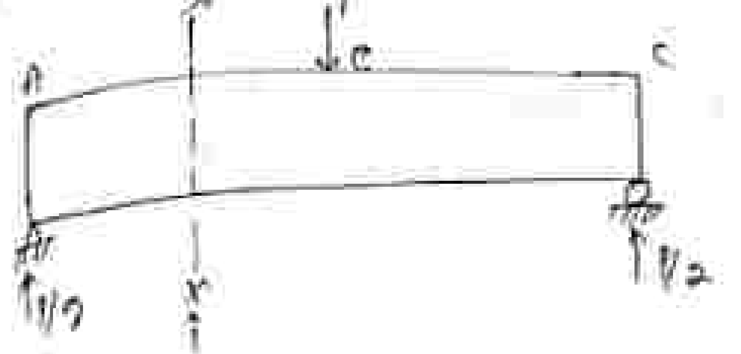
3) Uniformly varying load

Shear Force

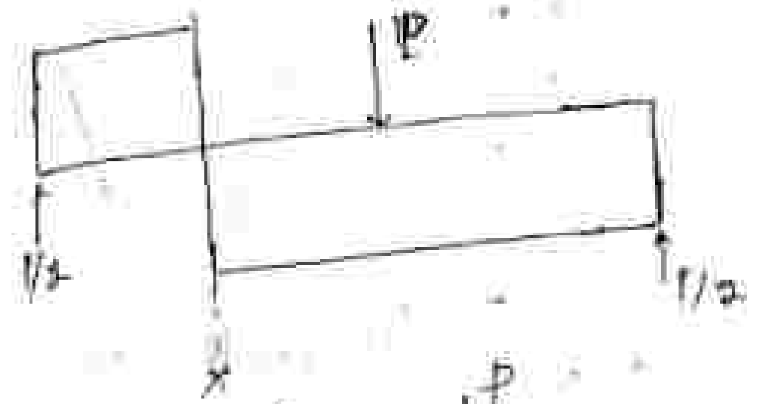
It is the internal resisting transverse force that required to convert a FBD into equilibrium either from the left of the section or from the right of the section.

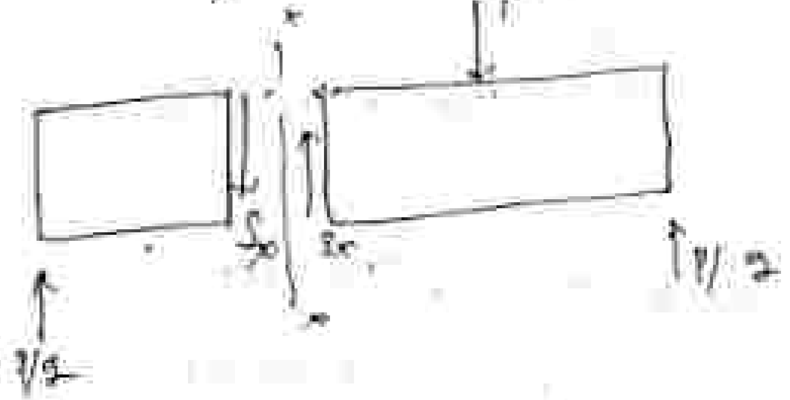
Alternatively it can be say that it is defined as the summation of all the transverse forces (unbalanced vertical force) (vertical force) either from left or from the right of the section (S.F)





$$SF = S_x$$
$$S_x = \frac{P}{2}$$

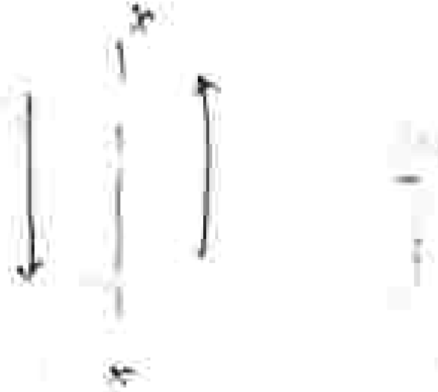




Sign convection :-

(+ve SF)

Left side upward
& Right side downward

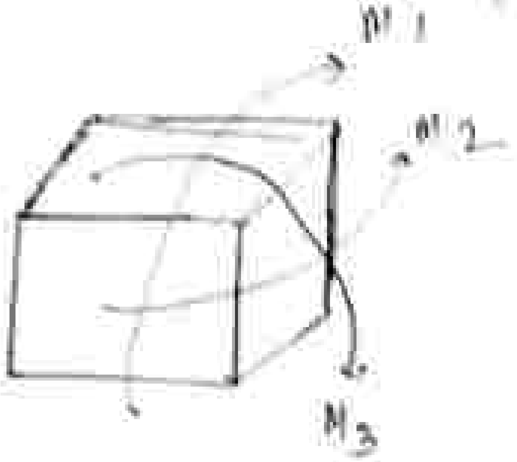


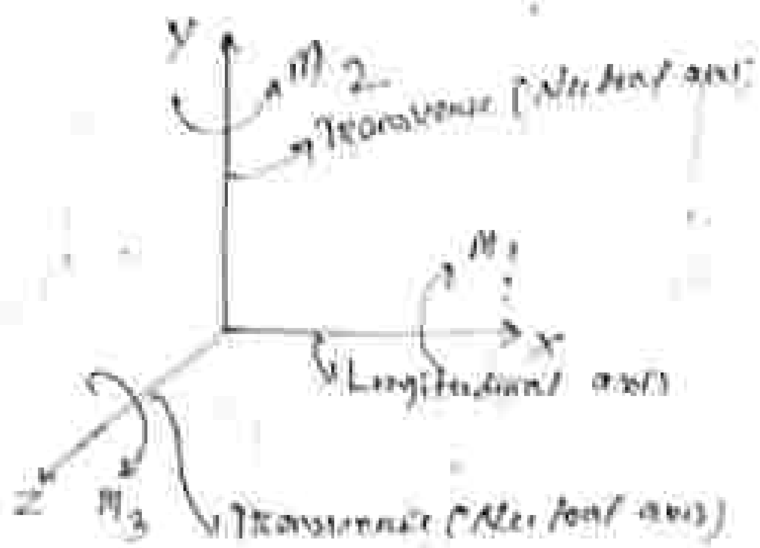
(-ve SF)

Left side downward &
Right side upward.

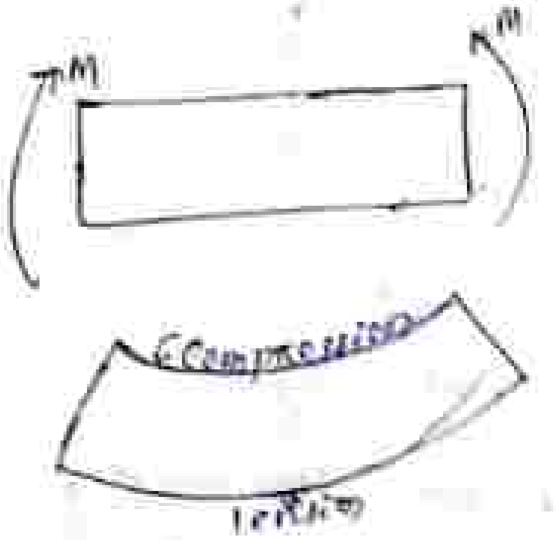
Bending moment :-

It is the resultant moment on any section either from the left or from the right of that section.

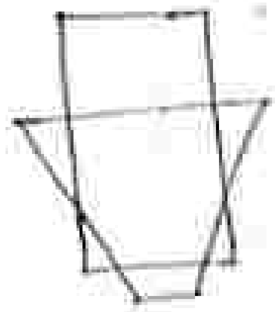




{ Couple about longitudinal axis : Torque
" " Transverse " : Moment }



(Front view)



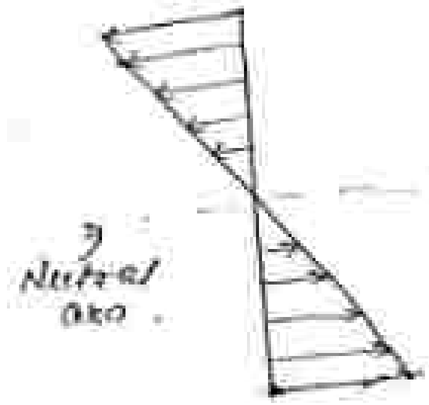
(S.V)

After bending if the top fibre of the beam gets compressed then the bottom fibre will be expanded and vice-versa.

If the crossection of the beam is rectangular before bending then it becomes trapezoidal after bending.

→ bending

$$\boxed{\frac{f(\sigma)}{y} = \frac{M}{I} = \frac{E}{R}}$$



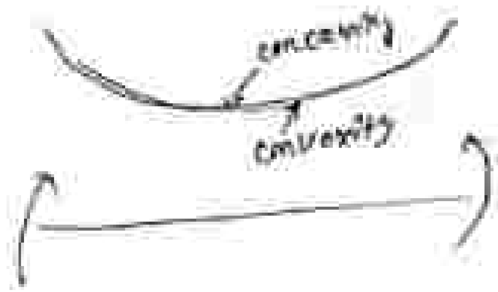
f = bending stress at y distance from neutral axis.

I = M I about neutral axis

R = Radius of curvature

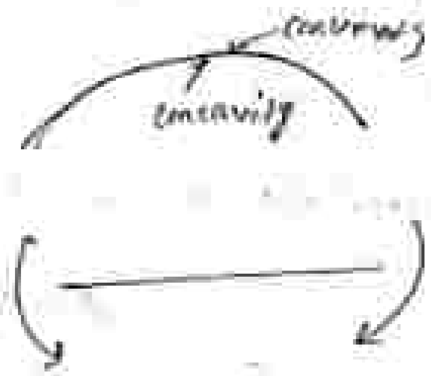
E = Youngs modulus

Sign convention for bending moment :-



Sagging or concave

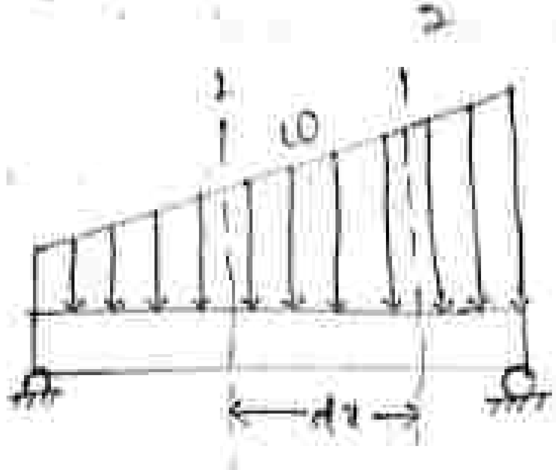
(+ve bending moment)

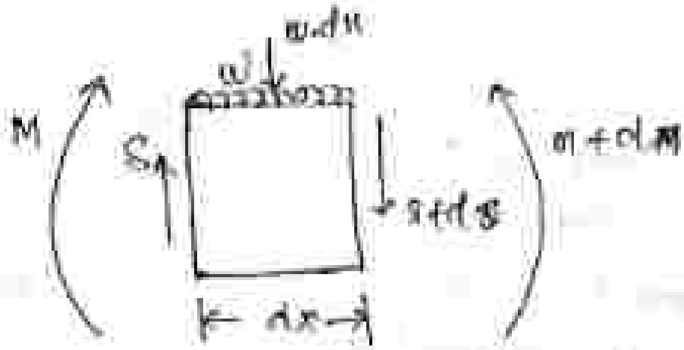


Hogging or concave

(-ve bending moment)

Relation b/w loading rate and shear force :-





Assumption :-

Let us assume that the shear force and bending moment are +ve on the elementary section.

For equilibrium in vertical ~~section~~ direction :-

$$S - S - dS - w\,dx = 0$$

$$\boxed{-\left(\frac{dS}{dx}\right) = w}$$

It means that the -ve slope of of curve at any section is equal to loading rate at that section.

Couple equilibrium equation about 1-1

$$M - M - dM + (s + ds)\, dx + w\, dx \cdot \frac{dx}{2} = 0$$

$$\Rightarrow -dM + s\,dx + ds\,dx + w\,dx \times \frac{dx}{2} = 0$$

By neglecting order,

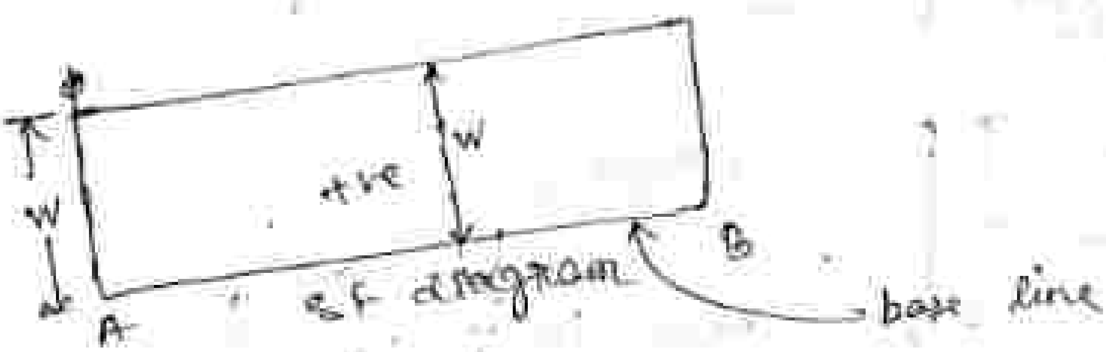
$$-dM + s\,dx = 0$$

$$\boxed{\frac{dM}{dx} = s}$$

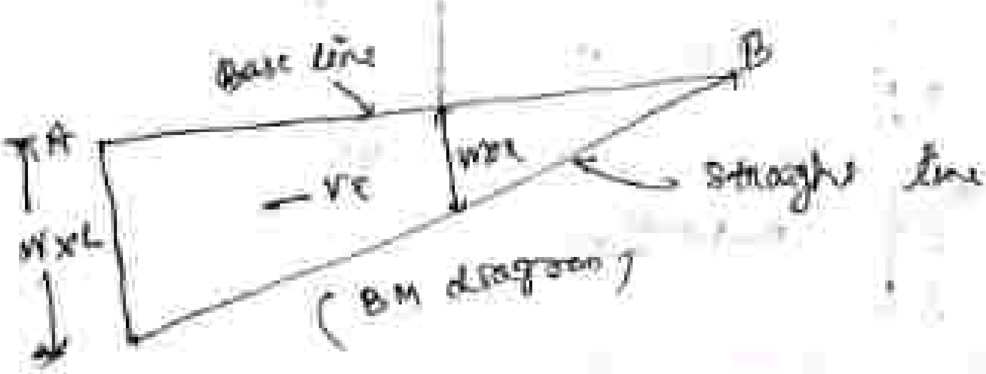
It means that the slope of BM curve at any section is equal to the magnitude of SF at that section.

* Cantilever Beam with a point load at its free end

(a)



(b)



(c)

Shear force at any section X, at a distance x from the free end, is equal to the total unbalanced vertical force.

i.e. $F_x = +W$ (+ve sign due to right down)

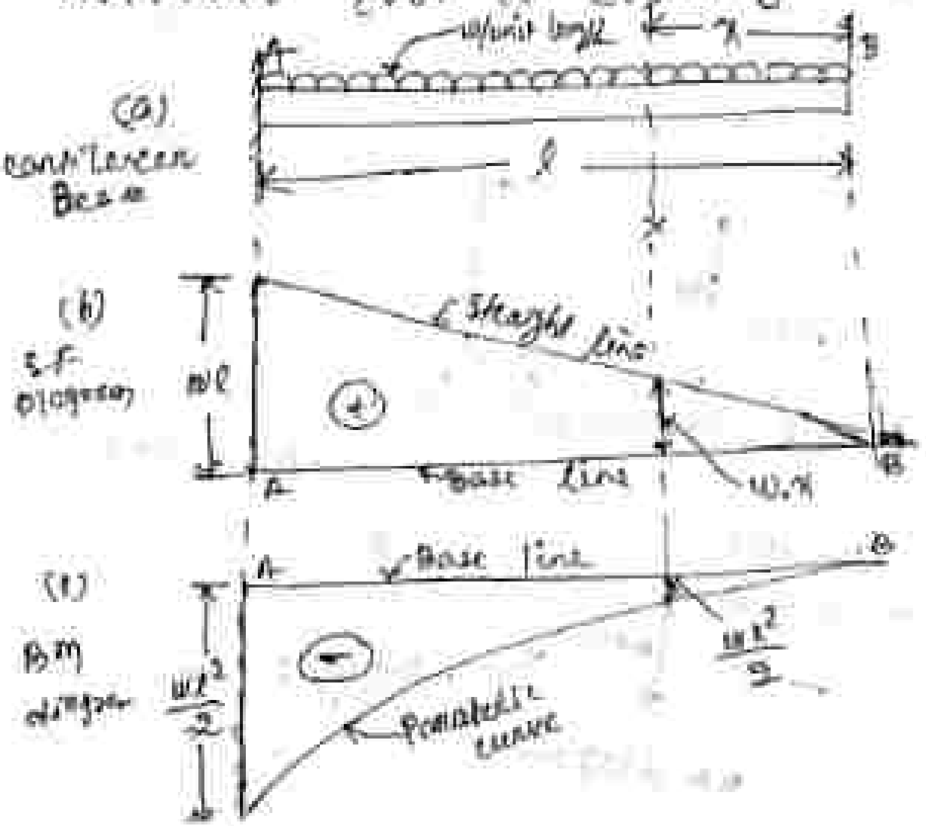
& bending moment at this section,

$M_x = -W \cdot x$ (-ve due to hogging)

Thus from the equation of shear force, we see that the shear force is constant and is equal to +W at all sections between B & A.

And from the bending moment equation, we see that the bending moment is zero at B ($x = 0$) and increases by a straight line law to $-Wl$ at A ($x = l$).

*** Cantilever with a uniformly distributed load**



shear force at any section x at a distance of x from B.

$F_x = +wx$

$F_x = 0$ (at $x=0$), $F_x = wl$ at ($x=l$)

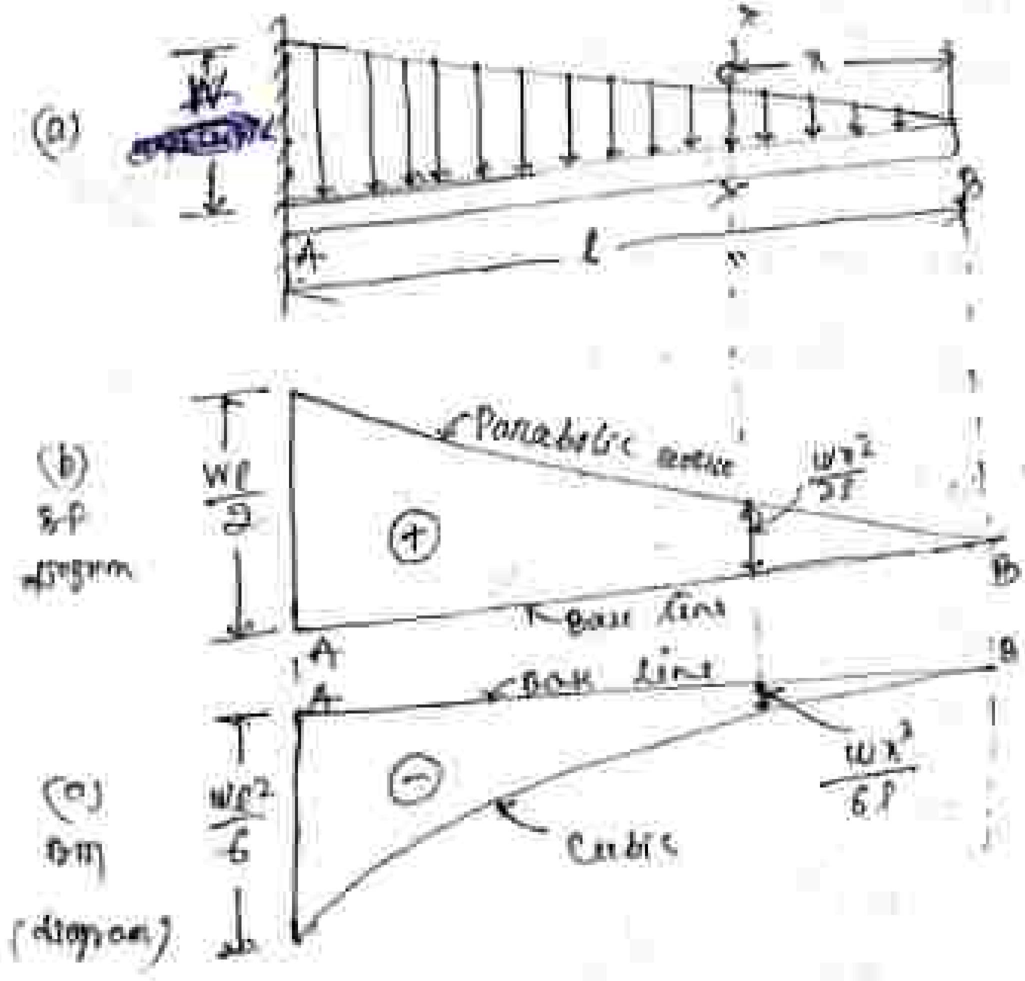
$100 - \frac{70}{111} \times 30$

Bending moment at x-x

$M_x = -w \cdot x \cdot \frac{x}{2} = -\frac{wx^2}{2}$

$M_x = 0$ (at $x=0$), $M_x = \frac{wl^2}{2}$ (at $x=l$)

* Cantilever Beam with gradually varying load



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Shear force at any section x-x, at a distance x from the free end B.

Let F_x = SF at the section x &

M_x = BM at the section x.

Let w represent the loading rate at the section x.

The rate of loading is zero at B and is w per meter run at A. This means that rate of loading for a length L is w per unit length. Hence rate of loading for a length of x will be $\frac{w}{L} \times x$ per unit length. Hence $Cx = \frac{w \cdot x}{L}$

The shear force at the section x at a distance x from free end is given by

F_x = Total load on the cantilever for a length x from the free end B.

= Area of triangle BCx

$\boxed{\begin{array}{l} 1 - w/ \\ 1 - \frac{w}{L} x^2 \end{array}}$

$$= \frac{1}{2}(xB \cdot x \times Cx) = \frac{x \times \left(\frac{w \cdot x}{L}\right)}{2}$$

$$= \frac{wx^2}{2L}$$

at $x = 0$, $F_x = 0$

at $x = L$, $F_x = \frac{wL^2}{2L} = \frac{wL}{2}$

The bending moment at the section x at a distance x from the free end B is given by,

$M_x = -$(Total load for a length x) $\times$ (Distance of the load from x)

$= -$(Area of triangle BCx) $\times$ Distance of c.g of the triangle from x.

$$= -\left(\frac{wx^2}{2L}\right) \times \frac{x}{3} = -\frac{wx^3}{6L}$$

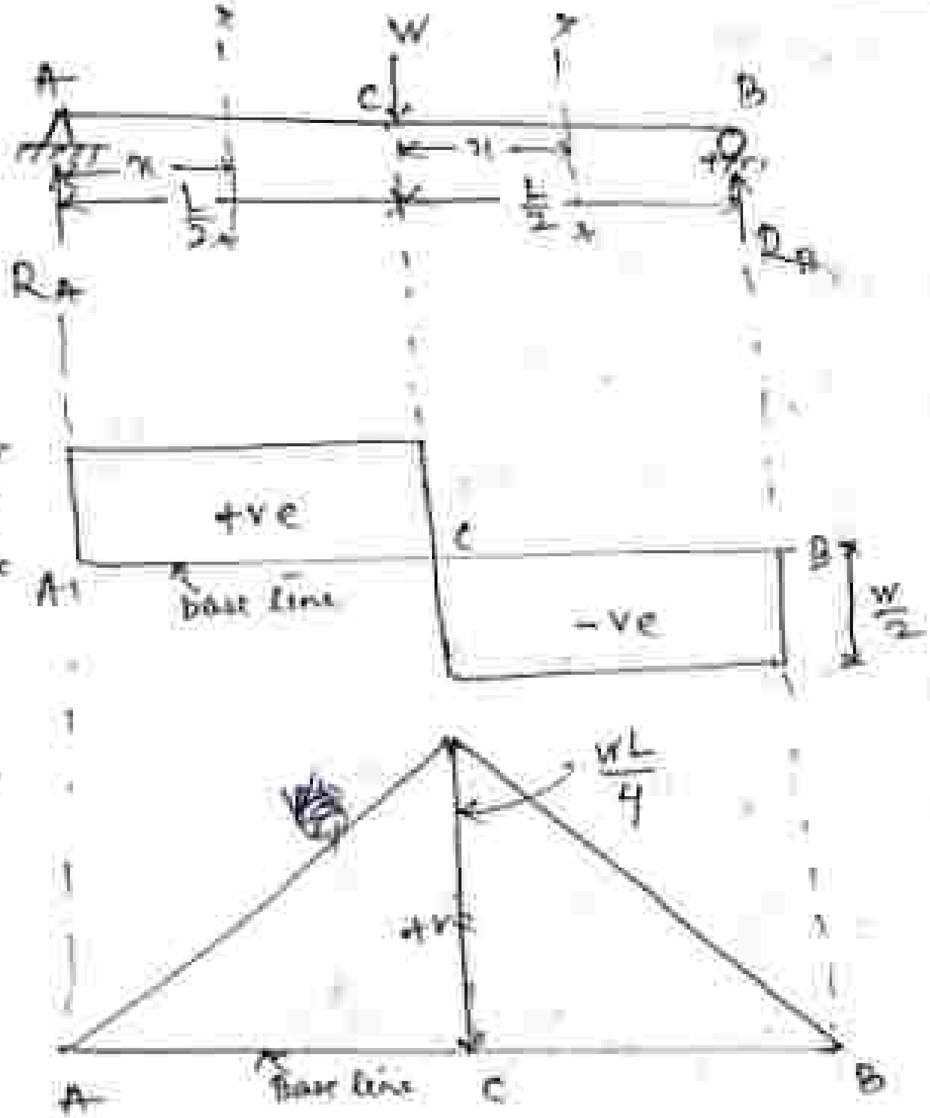
At B, $x = 0$ (BT), $M_B = 0$

At A, $x = L$, $M_A = -\frac{wL^3}{6L} = -\frac{wL^2}{6}$

(a) simply supported Beam

(b) SF diagram

(c) BM diagram



$R_A + R_B = W$

$M_A = 0$

$R_B \times l - W \times \frac{L}{2} = 0$

$\Rightarrow \boxed{R_B = \frac{W}{2}}$, $\boxed{R_A = \frac{W}{2}}$

SF in AC (x from A)

$S_x = R_A = \frac{W}{2}$

$S_A = S_B = \frac{W}{2}$

SF in BC (x from B)

$S_x = R_A - W = \frac{W}{2} - W = -\frac{W}{2}$

$C_B = S_x = -\frac{W}{2}$

BM in AC (x from A)

$M_x = R_A \times x = \frac{W}{2} x$

$M_A = \frac{W}{2}(0) = 0$, $M_C (x = \frac{L}{2}) = \frac{W}{2} \cdot \frac{L}{2} = \frac{WL}{4}$

BM in CB (x from C)

$M_x = R_A (x + \frac{L}{2}) - Wx$

$= \frac{W}{2} (\frac{L}{2} + x) - Wx$

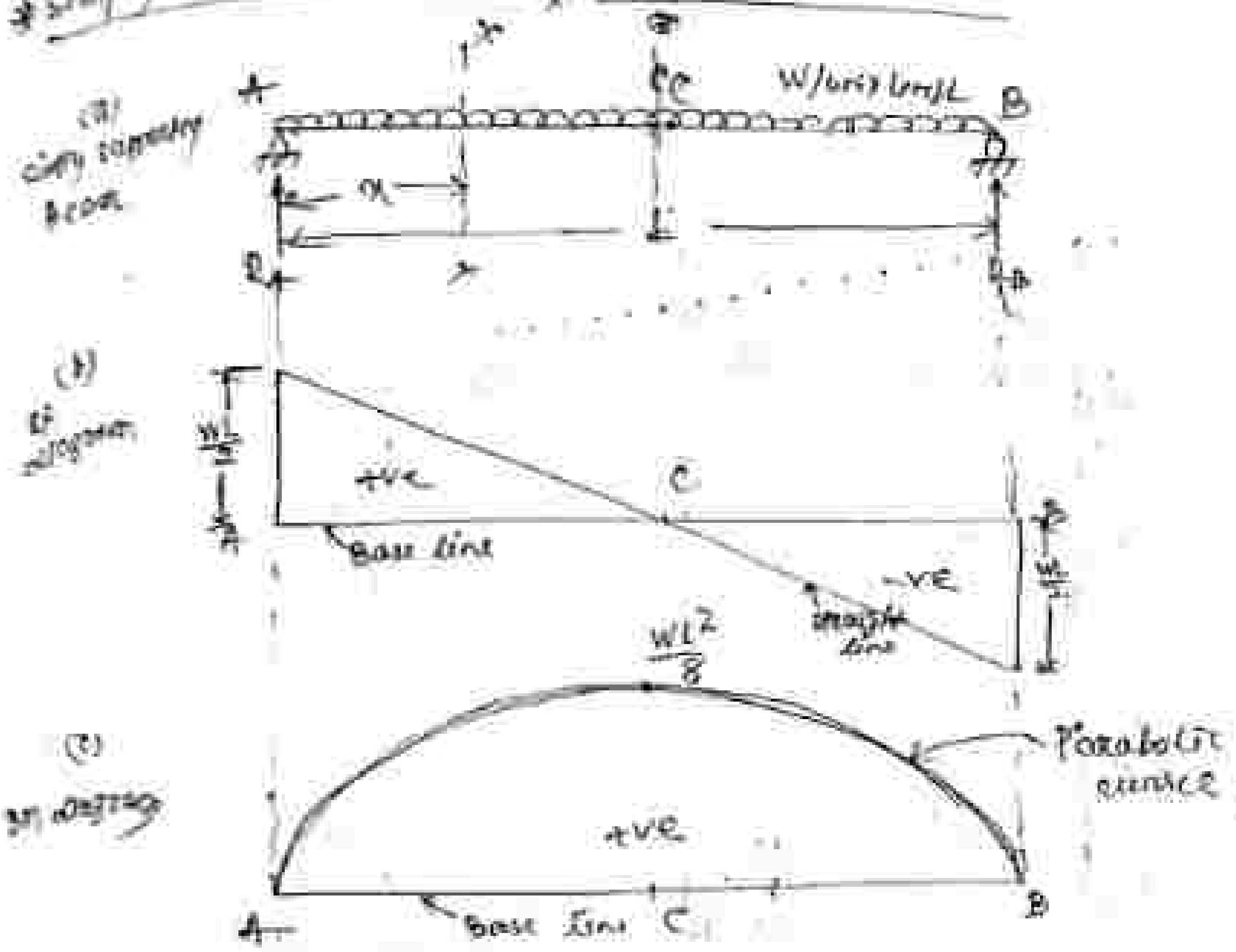
$= \frac{WL}{4} + \frac{Wx}{2} - Wx$

$= \frac{WL}{4} - \frac{Wx}{2}$

$M_C (x = 0) = \frac{WL}{4}$

$M_B (x = \frac{L}{2}) = \frac{WL}{4} - \frac{WL}{4} = 0$

* Simply supported beam with u.d.l



$R_A + R_B = wL$

$M_A = 0$

$\Rightarrow R_B \times L - wL \times \frac{L}{2} = 0$

$\boxed{R_B = R_A = \frac{wL}{2}}$

SF in AB (x from A)

$S_x = R_A - wx = \frac{wL}{2} - wx$

$S_A (x=0) = \frac{wL}{2}$

$S_C (x = \frac{L}{2}) = \frac{wL}{2} - w\frac{L}{2} = 0$

$S_B (x=L) = \frac{wL}{2} - wL = -\frac{wL}{2}$

$\boxed{S_x = 0,\ x = \frac{L}{2}}$

BM in AB (x from A)

$M_x = R_A \times x - w \cdot x \times \frac{x}{2}$

$= \frac{wL}{2} \times x - \frac{wx^2}{2}$

$M_{A(x=0)} = 0$

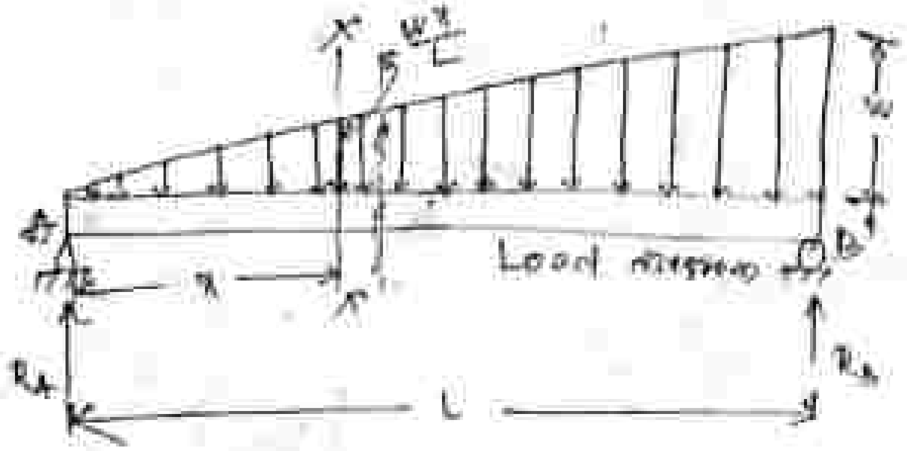
$M_{C(x=\frac{L}{2})} = \frac{wL}{2} \times \frac{L}{2} - \frac{wL^2}{8}$

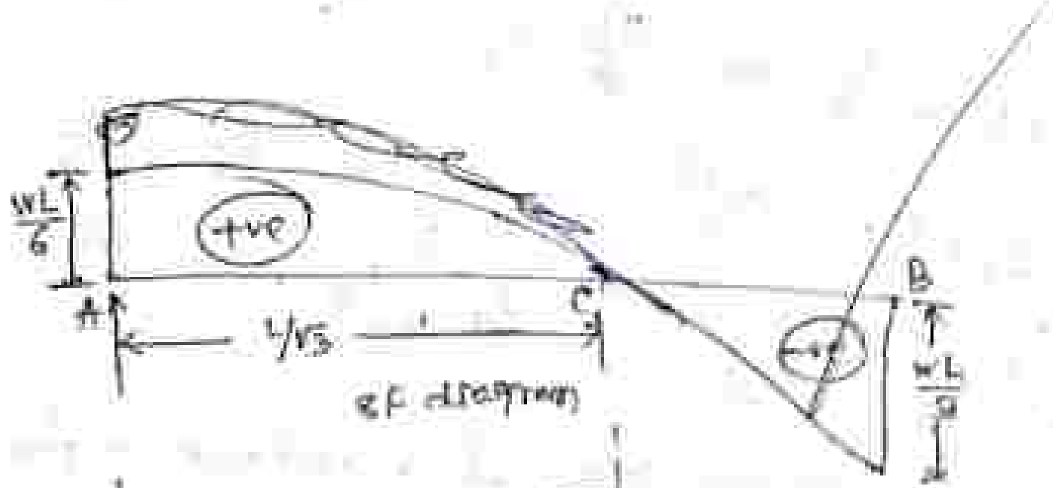
$= \frac{wL^2}{4} - \frac{wL^2}{8} = + \frac{wL^2}{8}$

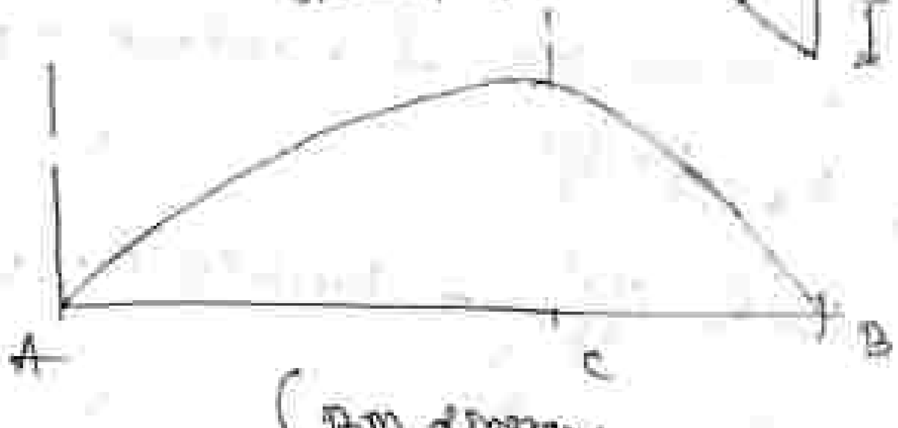
$M_{B(x=L)} = \frac{wL}{2} \cdot L - \frac{wL^2}{2}$

$= 0$

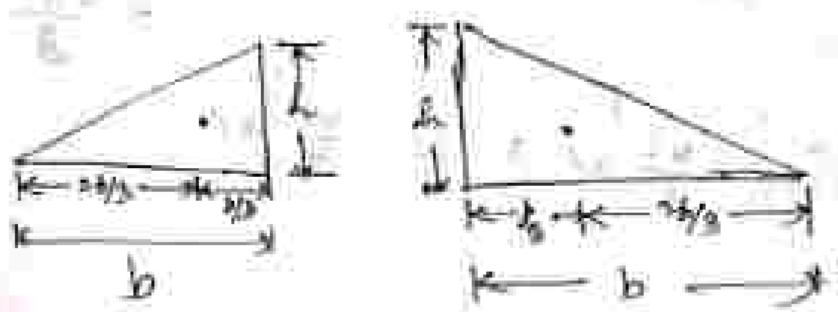
* Simply supported beam with a gradually varying load from zero at one end to W per unit length at the other







(BM diagram)



$R_A + R_B = \frac{wL}{2}$

$\Sigma M_A = 0$

$\Rightarrow R_B \times L - \frac{wL}{2} \times \frac{2L}{3} = 0$

$\Rightarrow \boxed{R_B = \frac{wL}{3}}$

$R_A = \frac{wL}{2} - \frac{wL}{3}$

$= \frac{3wL - 2wL}{6}$

$\boxed{R_A = \frac{wL}{6}}$

SF at any section x-x from distance x from A.

$S_x = R_A - \frac{1}{2} x \times \frac{wx}{L}$

$= \frac{wL}{6} - \frac{wx^2}{2L}$

At A, $(x = 0)$, $SF_A = \frac{wL}{6}$

At B, $(x = L)$, $SF_B = \frac{wL}{6} - \frac{wL^2}{2L} = \frac{wL - 3wL}{6}$

$= \frac{-2wL}{6} = -\frac{wL}{3}$

SF is $+\frac{wL}{6}$ at A & decreases to $-\frac{wL}{3}$ at B acc. to parabolic law. Hence SF will be zero at some where betn A & B.

$SF_x = 0$

$\Rightarrow \frac{wL}{6} - \frac{wx^2}{2L} = 0 \Rightarrow x = \frac{L}{\sqrt{3}} = 0.577L$

BM Diagram

$M_A = M_B = 0$

BM at the secn x-x at a distance x from the end A.

$M_x = R_A x - \text{Load on } Ax \times \frac{x}{3} = \frac{wL}{6} \cdot x - \frac{1}{2} x \times \frac{wx}{L} \times \frac{x}{3}$

$= \frac{wL}{6} x - \frac{wx^3}{6L}$ ← cubic law

Max. BM occurs at a point where SF becomes zero after changing its sign. That point is at a distance $\frac{L}{\sqrt{3}}$ from A.

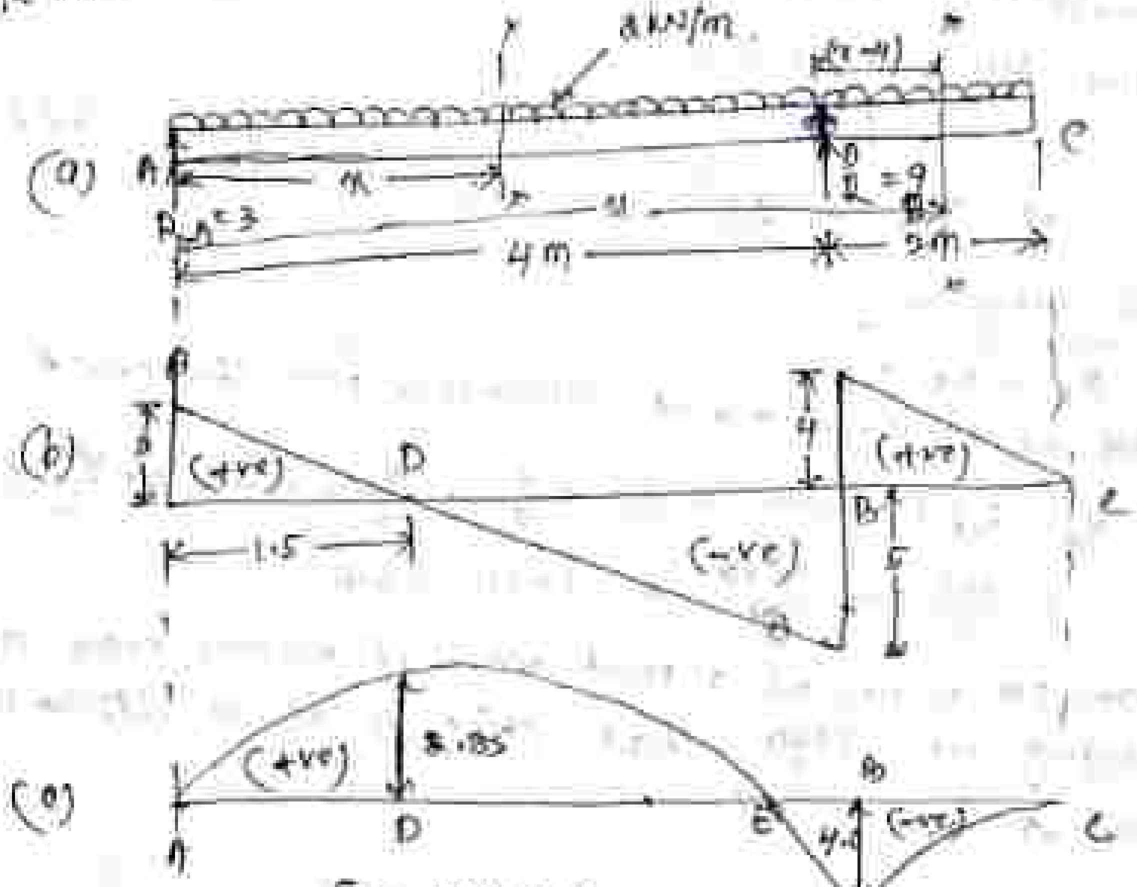
$$\text{Now, } M_{(x=\frac{L}{\sqrt{3}})} = \frac{wL}{6}\cdot\frac{L}{\sqrt{3}} - \frac{w}{6L}\left(\frac{L}{\sqrt{3}}\right)^3$$

$$= \frac{wL^2}{6\sqrt{3}} - \frac{wL^2}{18\sqrt{3}} = \frac{3wL^2 - wL^2}{18\sqrt{3}} = \frac{wL^2}{9\sqrt{3}}$$

*** Shear force and Bending Moment Diagram for over-hanging beam.**

If the end portion of a beam is extended beyond the support, such beam is known as overhanging beam. In case of overhanging beams, the BM is positive b/w the two supports whereas BM is -ve for overhanging portion. Hence at some point, the BM is zero after changing its sign from +ve to -ve or vice versa. That point is known as point of contraflexure or point of inflexion.

Problem :- Draw SF & BM dia. & find point of inflexion.



(BM Diagram)

Sol[n]:- $R_A + R_B = 2 \times 6 = 12\ kN$

$M_A = 0$

$\Rightarrow R_B \times 4 - 2 \times 6 \times \frac{6}{2} = 0$

$\Rightarrow \boxed{R_B = \frac{12}{4} = 9\ kN}$, $\boxed{R_A = 3\ kN}$

Shear Force Diagram

SF at $A = S_A = +R_A = +3\ kN$

i) SF at any secn x-x b/w A & B at a distance x from A

$S_x = R_A - 2x$ ← straight line law

$S_{A(x=0)} = +R_A = +3\ kN$

$S_{B(x=4m)} = 3 - 2 \times 4 = 3 - 8 = -5\ kN$

Let, $S_x = 0$

$\Rightarrow 3 - 2x = 0 \Rightarrow \boxed{x = \frac{3}{2} = 1.5\ m}$

ii) SF at any secn b/w B & C at a distance x from A

$S_x = +R_A - 2 \times 4 + R_B - 2(x-4)$

$= 3 - \cancel{8} + 9 + \cancel{8} - 2x = 12 - 2x$

$S_{B(x=4)} = 12 - 2 \times 4 = 12 - 8 = +4\ kN$.

$S_{C(x=6m)} = 12 - 2 \times 6 = 12 - 12 = 0\ kN$.

Bending moment diagram

$M_A = 0$

i) The BM at any secn b/w A & B at dist x from A,

$M_x = R_A \times x - 2x \times \frac{x}{2} = 3x - x^2$ ← Parabolic law

$M_{A(x=0)} = 0$, $M_{B(x=4)} = 12 - 16 = -4\ kN\,m$

Max. BM occurs at $x = 1.5$ m

$M_{(x=1.5)} = 3 \times 1.5 - (1.5)^2 = 4.5 - 2.25 = 2.25$ kNm

(ii) The BM at any section b/w B & C at a dist. x from A.

$M_x = R_A \times x - 2 \times x \times \frac{x}{2} + R_B (x-4)$

$= 3x - x^2 + 9(x-4)$

At B, $x=4$, $M_B = 3\times4 - 4^2 + 9(0) = -4$ kNm

At C, $x=6$, $M_C = 3\times6 - 6^2 + 9(6-4) = 0$

Point of contraflexure :-

$M_x = 0$

$\Rightarrow 3x - x^2 = 0$

$\Rightarrow x = 3$

Hence point of contraflexure will be at a distance of 3m from A.

* For simply supported beam

(i) If loading is concentrated load (point load) and BMD will be rectangular (uniform) and SFD will be triangular (linear).

(ii) If the loading is u.d.l, the SFD will be triangular and BM diagram will be parabolic (2nd order)

(iii) If the loading is u.v.l, the SFD will be parabolic and BM diagram will be cubic (3rd order).

(iv) If the shear force changes its sign at a section then bending moment will be maximum at that point in that section.

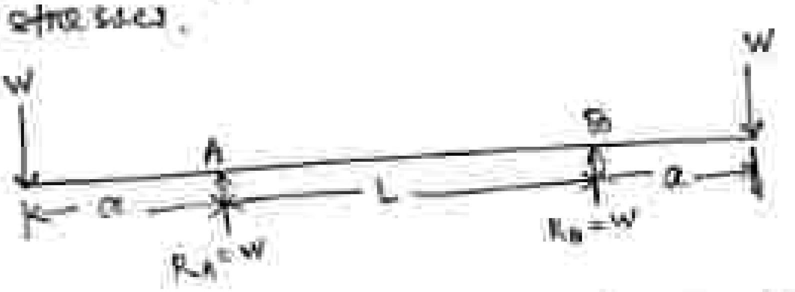
(v) If the BM changes its sign at a point then curvature also changes in sign at that point and such point is known as point of contraflexure or point of inflection. (BM = 0)

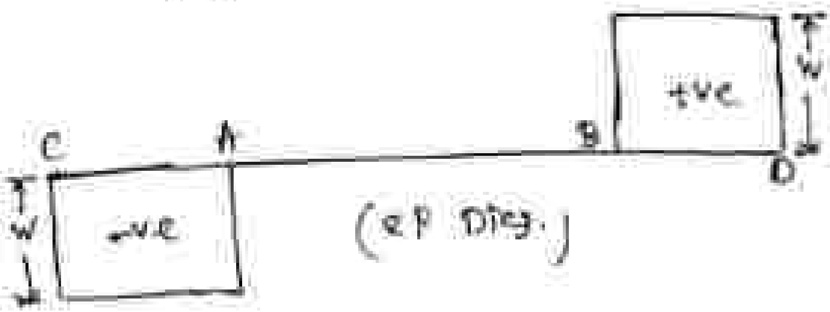
THEORY OF SIMPLE BENDING

When some external load acts on a beam, the shear forces and bending moments are set up at all sections of the beam. Due to SF & BM, the beam undergoes certain deformations. The material of the beam will offer resistance or stress against these deformations. These stress with certain assumptions can be calculated. The stress introduced by bending moment are known as bending stresses.

Pure Bending or Simple Bending:-

If a length of beam is subjected to a constant bending moment and no shear force (i.e zero shear force), then the stresses will be set up in that length of the beam due to BM only and that length of the beam is said to be in pure bending or simple bending. The stresses set up in that length of beam are known as bending stresses.





(SF Diag.)

(BM Diag.)

There is no shear force b/w A & B but the BM

b/w A & B is constant.

This means that b/w A & B, the beam is subjected to a constant BM only. This condition of the beam b/w A & B is known as pure bending or simple bending.

Assumptions in the Theory on Simple Bending

1) The material of the beam is homogeneous (material is of same kind) and isotropic (elastic properties same in all direcn)

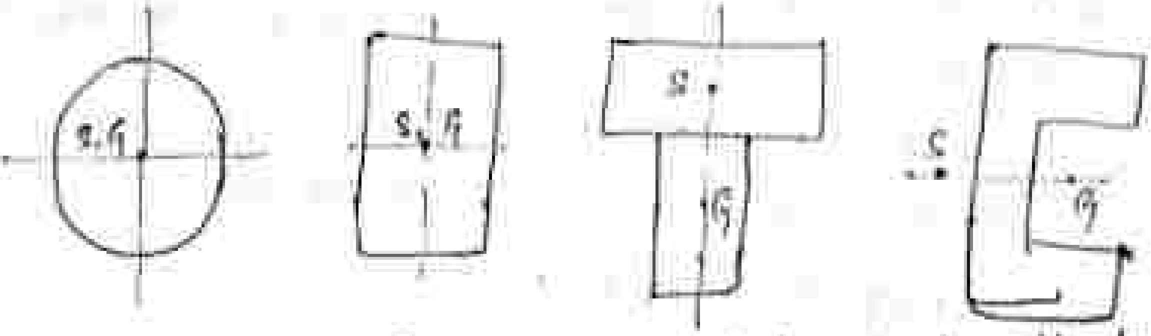
2) The value of Young's modulus of elasticity remains same in tension and compression.

3) The beam material is stressed within elastic limit and thus obeys the Hooke's law.

4) The transverse sections which were plane before bending remain plane after bending. It means that distribution of bending strain is linear from zero at neutral axis to max. at the surface.

5) For simple bending, section should be symmetrical in the plane of loading. It means that the load should pass through shear centre.

Shear centre is that point through which if load is passes then there will not be any twisting and only bending occurs.



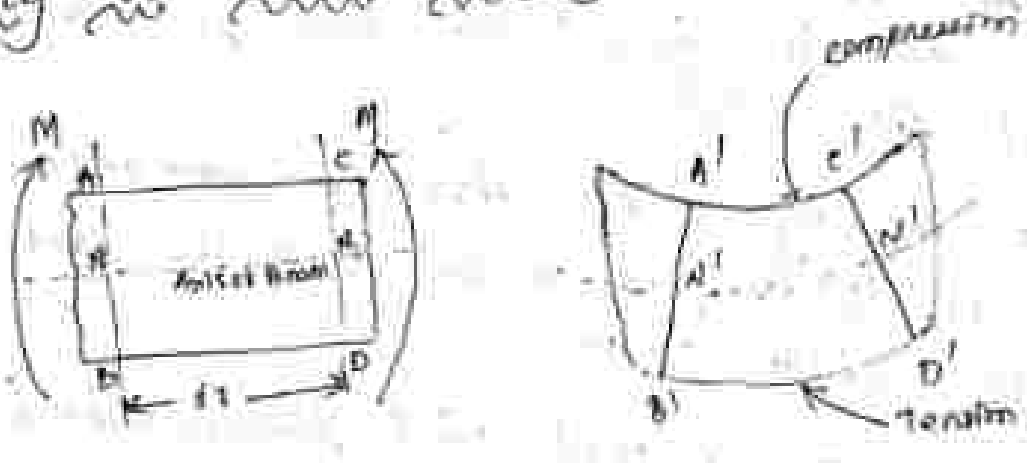
(6) The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.

(7) The radius of curvature is large compared with the dimensions of the cross section.

(8) Each layer of beam is free to expand or contract independently of the layer, above or below it.

(9) The beam is in equilibrium i.e there is no resultant pull or push in the beam section.

Theory of simple Bending:-



(Before Bending)

→ Top layer shortened in its length

→ Bottom layer will expand or elongated.

→ B/w them neither shortened nor elongated. That layer is known as neutral layer or ~~neutral axis~~ or neutral surface.

The line of intersection of the neutral layer on a cross section of a beam is known as neutral axis (N.A).

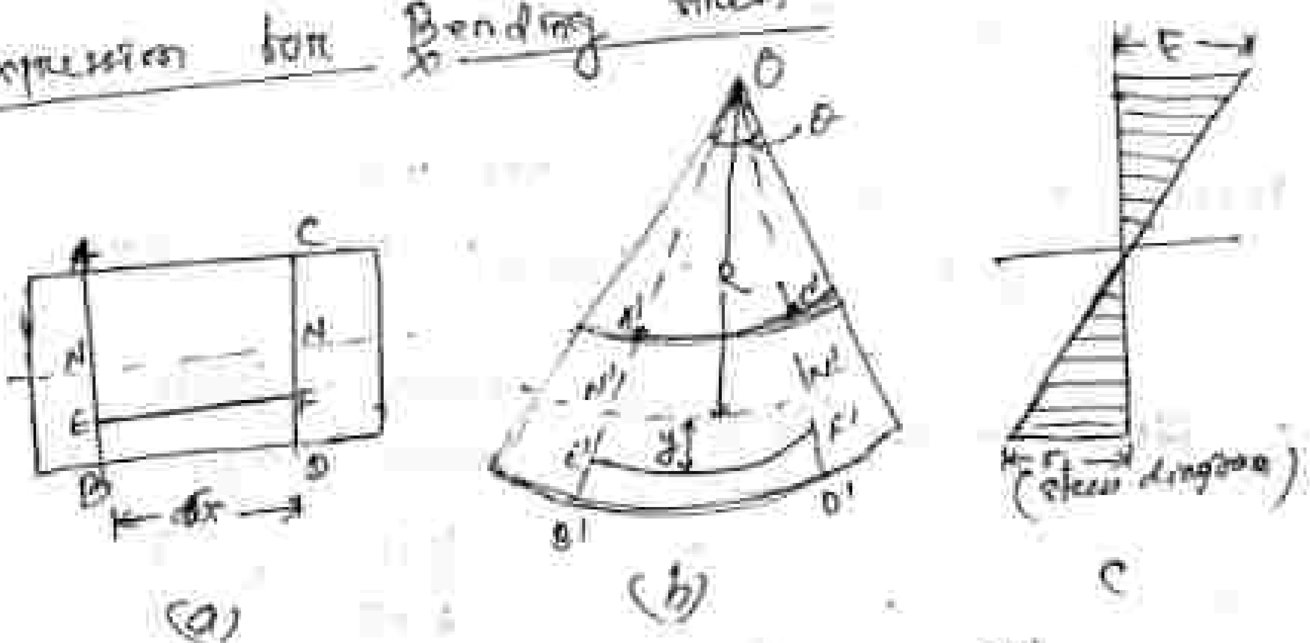
The layers above N-N (or N'-N') have been shortened.

Due to decrease in lengths of the layers above N-N, will be subjected to comp.

Top layer has been shortened maximum. As we proceed towards the layer N-N, the ~~decrease~~ in length of the layers decreases.

At the layer N-N, there is no change in length. This means comp. stress will be max. at top layer.

Expression for Bending stress



Let R = Radius of neutral layer N'-N'

θ = Angle subtended at O by N'B' & C'D' produced

Strain variation Along the Depth of Beam

Original length of layer $EF = \delta x$

Also " " neutral layer, i.e. $NN = \delta x$

After bending, $N'N' = NN = \delta x$ but $EF \neq E'F'$

Now from fig. (b)

$$N'N' = R \times \theta$$

$$\text{and } E'F' = (R+y) \times \theta$$

But $N'N' = NN = \delta x$

$$\therefore \delta x = R \times \theta$$

Increase in length of the layer EF

$$= E'F' - EF = (R+y) \times \theta - R \times \theta = y \times \theta$$

$$\therefore \text{Strain in the layer } EF = \frac{\text{Increase in length}}{\text{Original length}}$$

$$= \frac{y \times \theta}{EF} = \frac{y \times \theta}{R \times \theta} = \frac{y}{R}$$

$$\therefore \boxed{e = \frac{y}{R}}$$

As R is constant, strain in a layer is proportional to its distance from the neutral axis.

Variation of strain is linear.

Stress variation :

Let σ = Stress in the layer EF

E = Youngs modulus of the beam

Then E = $\dfrac{\text{stress in the layer EF}}{\text{strain in the layer EF}}$

$$\therefore E = \frac{\sigma}{\left(\frac{y}{R}\right)}$$

$$\therefore \boxed{\sigma = E \times \frac{y}{R} = \frac{E}{R} \times y}$$

Stress in any layer is directly proportional to the distance of the layer from the neutral axis.

$$\boxed{\therefore \frac{\sigma}{y} = \frac{E}{R}}$$

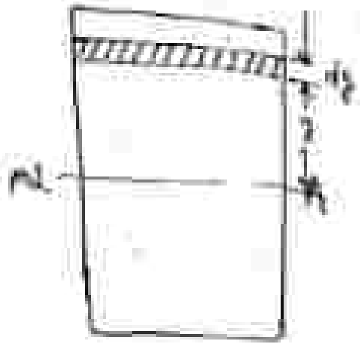
Position of Neutral Axis

The line of intersection of the neutral layer, with any normal cross-section of a beam is known as neutral axis of that section.

on transverse section

stress at a distance y from the neutral axis is given by,

$$\sigma = \frac{E}{R} \times y$$



Let dA be the area of the section

Now force on the layer

$=$ stress on layer $\times$ area of layer

$$= \sigma \times dA = \frac{E}{R} \times y \times dA$$

$\therefore$ Total force on the beam section

$$= \int \frac{E}{R} \times y \times dA$$

$$= \frac{E}{R} \int y \times dA$$

But for pure bending, there is no force on the section of the beam (or force is zero)

$$\therefore \frac{E}{R} \int y \times dA = 0$$

$$\Rightarrow \int y \times dA = 0 \qquad \left(\frac{E}{R} \text{ cannot be zero}\right)$$

Now $y \times dA$ represents the moment of area dA about neutral axis. Hence $\int y \times dA$ represents the moment of entire area of the section about neutral axis. But we know that moment of any area about an axis passing through its centroid, is equal to zero. Hence neutral axis coincides with the centroidal axis. Thus centroidal axis of a section gives the position of neutral axis.

Moment of Resistance

Due to pure bending, the layers above the N.A. are subjected to C.S. whereas the layers below N.A. are subjected to T.S. Due to these stress, the forces will be acting on the layers. These forces will have moment about the N.A. The total moment of these forces about the N.A. for a section is known as moment of resistance of that section.

The force on the layer at a distance y from neutral axis $= \frac{E}{R} \times y \times dA$

Moment of this force about N.A

$= \text{Force on layer} \times y$

$= \frac{E}{R} \times y \times dA \times y$

$= \frac{E}{R} \times y^2 \times dA$

Total moment of the forces on the section of the beam (or moment of resistance) $= \int \frac{E}{R} \times y^2 \times dA = \frac{E}{R} \int y^2 \times dA$

let M = External moment applied on the beam section.

For equilibrium, $M = \frac{E}{R} \int y^2 \times dA$

But $\int y^2 \times dA$ represents the moment of inertia of the area of the section about the neutral axis. Let it be I.

$\therefore M = \frac{E}{R} \times I$ or $\frac{M}{I} = \frac{E}{R}$

But $\frac{\sigma}{y} = \frac{E}{R}$,

Hence, $\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$ ← bending equation

M = moment of resistance
I = MI of the secⁿ about N.A.
R = Radius of curvature of N.A.
σ = bending stress

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Units $M = N\text{-}mm$, $I = mm^4$

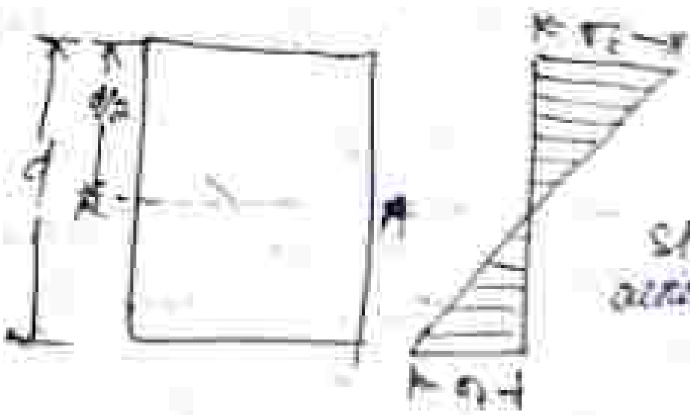
$\sigma = N/mm^2$, $y = mm$

$E = N/mm^2$, $R = mm$

Condition of simple bending:

The eqn $\boxed{\dfrac{M}{I} = \dfrac{\sigma}{y} = \dfrac{E}{R}}$ is applicable to a member which is subjected to a constant BM and the member is absolutely free from shear force. But in actual practice BM varies from section to section and also shear force is not zero. But shear force is zero at a section where BM is maximum. Hence the condition of simple bending may be assumed to be satisfied at such a section.

*Bending stress in symmetrical sections



stress distribution across a section

Section Modulus :-

Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It is denoted by the symbol Z.

Mathematically, $Z = \frac{I}{y_{max}}$

Where I = M.O.I about neutral axis

y_{max} =

We have $\frac{M}{I} = \frac{\sigma}{y}$

σ will be maximum, where y is maximum. Hence above equation can be written as

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$\therefore M = \sigma_{max} \cdot \frac{I}{y_{max}}$$

$$\text{or } \boxed{M = \sigma_{max} Z}$$

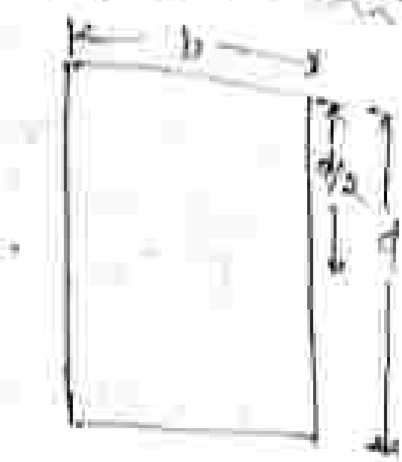
where M = maximum bending moment (or moment of resistance offered by the section)

Hence moment of resistance offered the section is max. when section modulus Z is maximum. Hence section modulus represent the strength of the section

* Section modulus for various shapes or beam sections

~~MI of a~~

(1) Rectangular Section



MI of a rectangular section about an axis through its C.G (or N.A) is given by

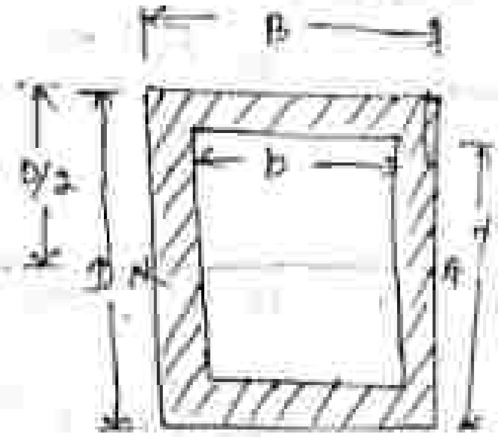
$$I = \frac{bd^3}{12}$$

Distance of outermost layer from N.A is given by

$$y_{max} = \frac{d}{2}$$

$$\therefore Z = \frac{I}{y_{max}} = \frac{bd^3}{12 \times \left(\frac{d}{2}\right)}$$

$$= \frac{bd^3}{6 \times d} = \frac{bd^2}{6}$$



(2) Hollow Rectangular Section

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{1}{12}(BD^3 - bd^3)$$

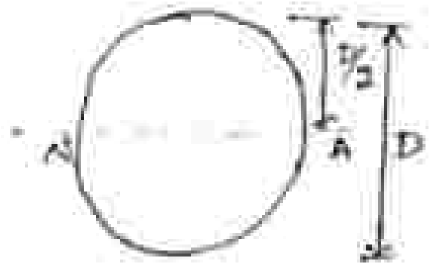
$$y_{max} = D/2$$

$$\therefore Z = \frac{I}{y_{max}} = \frac{\frac{1}{12}\left[BD^3 - bd^3\right]}{\left(\frac{D}{2}\right)}$$

$$= \frac{1}{6D}\left[BD^3 - bd^3\right]$$

Circular Section

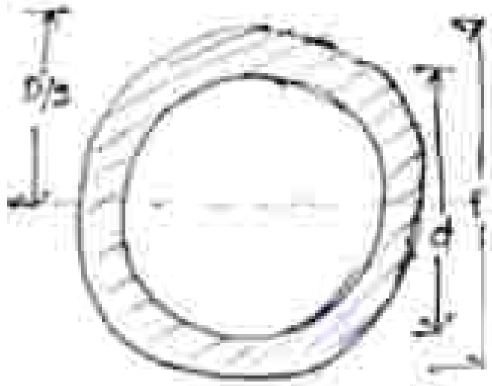
$$I = \frac{\pi D^4}{64} \quad \& \quad y_{max} = \frac{D}{2}$$



$$\therefore Z = \frac{I}{y_{max}} = \frac{\left(\frac{\pi D^4}{64}\right)}{\left(\frac{D}{2}\right)}$$

$$= \frac{\pi D^4}{64\left(\frac{D}{2}\right)} = \frac{\pi D^3}{32}$$

Hollow circular Section



$$I = \frac{\pi}{64}\left(D^4 - d^4\right)$$

$$y_{max} = \frac{D}{2}$$

$$\therefore Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64}\left[D^4 - d^4\right]}{\left(\frac{D}{2}\right)}$$

$$= \frac{\pi}{32D}\left[D^4 - d^4\right]$$

STRUT

Defⁿ:- A structural member, subjected to an axial compressive force, is called a strut. As per definition a strut may be horizontal, inclined or even vertical. Ex- Connecting rods, piston rods, rods in engine etc.

A structural member subjected to axial compressive force when used in trusses is known as strut. When it is used in cranes it is called boom.

But A vertical strut, used in building or frames is called column.

Failure of a column

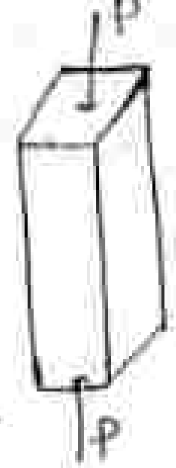
The failure of a column or strut takes place due to any one of the following stresses

(i) Crushing failure (due to direct compressive stress)
(ii) Buckling failure (due to buckling stresses)
(iii) Combined crushing and buckling failure

Generally short columns fails in crushing whereas long column fails in buckling. The intermediate columns may fail in the combined action.

Failure of a short column

Compressive stress $\sigma = \frac{P}{A}$



If this comp load is gradually increased, a stage will reach when the column will be on the point of failure by crushing. The stress induced in the column corresponding to this load is known as crushing stress and the load is called crushing load.

Let P_c = crushing load

σ_c = " stress

A = Area of cross-sec^n

Then $\boxed{\sigma_c = \frac{P_c}{A}}$

All short columns fail due to crushing.

Failure of long columns

Long columns don't fail by crushing alone, but also by bending (buckling).

The load at which the column just buckles, is known as buckling load, or critical ~~load~~ Just, or crippling load.

The buckling load is less than the crushing load for a long column.

Actually the value of buckling load for a long column is low whereas for short columns the value of buckling load is relatively high.

let P = comp. load at which the column just buckles

e = Max. bending of the column at the centre.

σ_o = stress due to direct load $= \frac{P}{A}$

σ_b = stress due to bending at the centre of the column $= \frac{P \times e}{Z}$

Where Z = sectⁿ modulus about the axis of bending.

The extreme stresses on the mid-section are given by

Max. stress $= \sigma_o + \sigma_b$

& Min. " $= \sigma_o - \sigma_b$

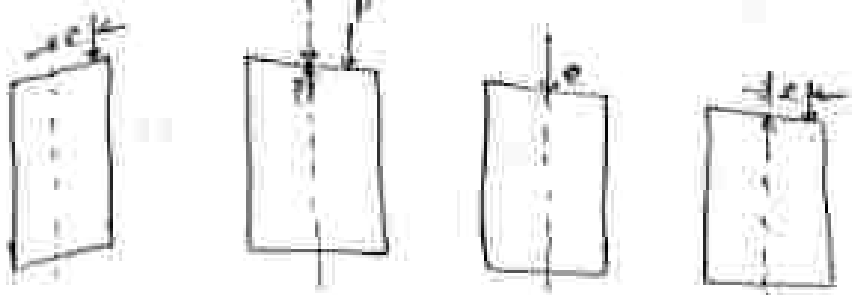
The column will fail when max. stress $(\sigma_o + \sigma_b)$ is more than crushing stress σ_c. But in case of long columns, the direct compressive stresses are negligible as compared to buckling stresses.

Hence very long columns are subjected to buckling stresses only.

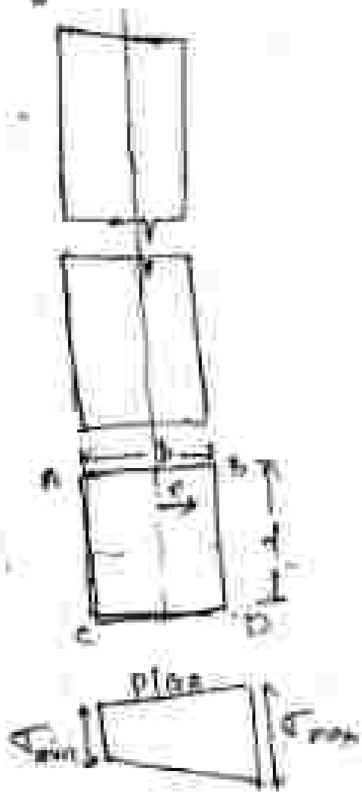
* Eccentric load in columns

Eccentric load :-

A load whose line of action does not coincide with the axis of column is called eccentric load.



Direct stress, bending stress, maximum & minimum stress.



Consider the above column ABCD subjected to an eccentric load about one axis (Y-Y axis)

Let ~~[illegible]~~ P = load acting on the column

e = eccentricity of the load

b = width of the section

d = thickness of the column.

Now Area of the section = $b \times d$

$$I = \cancel{\frac{bd^3}{12}} \quad \frac{db^3}{12}$$

$$Z = \frac{I}{y} = \cancel{\frac{bd^3/12}{}} = \frac{db^3}{12} /$$

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Direct stress, $\sigma_b = \frac{P}{A}$

Moment due to load, $M = P \cdot e$

Bending stress at any point of column section at a distance y from y-y axis

$$\sigma_b = \frac{M}{I} y = \frac{M}{Z}$$

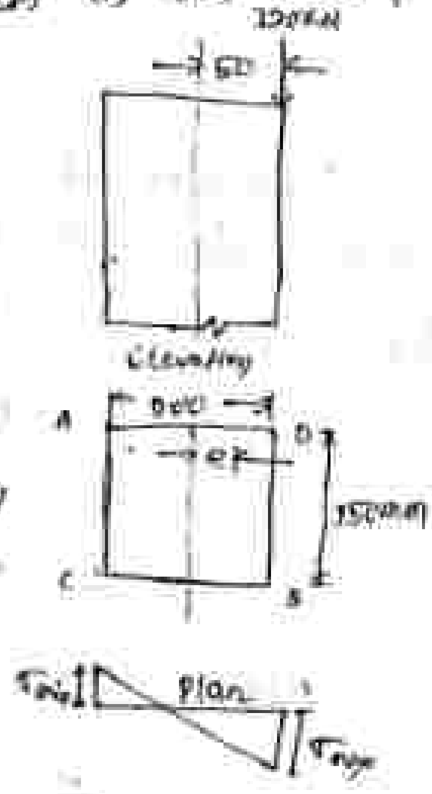
or at $y = \frac{b}{2}$, $\sigma_b = \dfrac{M \frac{b}{2}}{\frac{db^3}{12}} = \dfrac{6M}{db^2} = \dfrac{6P \cdot e}{db^2} = \dfrac{6Pe}{A \cdot b}$

Total stress = direct stress + bending stress

$$= \frac{P}{A} \pm \frac{M}{Z} = \frac{P}{A} \pm \frac{6Pe}{A \cdot b}$$

Problem

A rectangular column 200mm wide and 150mm thick is carrying a vertical load of 120 kN at an eccentricity of 50mm in a plane bisecting the thickness. Determine the max. & min. intensity of stress in the section.



Ans

Given: $b = 200mm$, $d = 150mm$,

$P = 120kN$, $e = 50mm$

Maximum stress

$$A = b \times d = 200 \times 150 = 30,000 \ mm^2$$

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{120 \times 10^3}{30,000}\left(1 + \frac{6 \times 50}{200}\right) = 10N/mm^2 = 10MPa \text{ (Ans)}$$

Minimum stress

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

$$= \frac{120 \times 10^3}{30,000}\left(1 - \frac{6 \times 50}{200}\right)$$

$$= -2MPa \text{ (tension)}$$

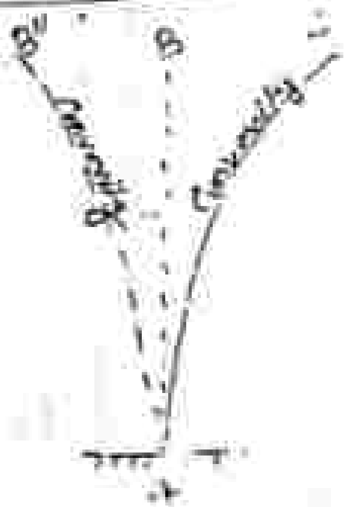
Assumptions made in the Euller's column Theory

1. The column is initially perfectly straight and the load is applied axially.
2. The cross-section of the column is uniform throughout its length.
3. The column material are perfectly elastic, homogeneous and isotropic & Obeys Hooke's Law.
4. The length of the column is very large as compared to its lateral dimensions.
5. The direct stress is very small as compared to the bending stress.
6. The column will fail by buckling alone.
7. The self weight of the column is negligible

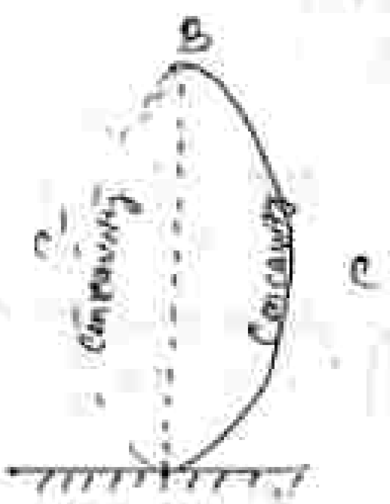
End condition for long columns

1. Both end hinged (or pinned)
2. One end hinged and one end is fixed.
3. Both end fixed.
4. One end fixed and other end free.

Sign convention:



(Positive)



(Negative)

Crippling Load and effective length

(a) Both end hinged (or pinned)

$$\boxed{L_e = L}$$ L_e = effective length

$$\boxed{P_e = \frac{\pi^2 EI}{L^2}}$$

where P_e = Euler's buckling load or crippling load

or $$\boxed{P_e = \frac{n^2 \pi^2 EI}{L^2}}$$

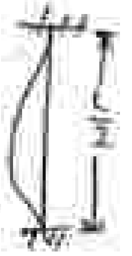
n = number of buckling loop

(b) One end hinged and one end fixed

$$\boxed{L_e = \frac{L}{\sqrt{2}}}$$

$$\boxed{P_e = \frac{2\pi^2 EI}{L^2}}$$

(c) Both end fixed



$$\boxed{L_e = \frac{L}{2}}$$

$$\boxed{P_e = \frac{4\pi^2 EI}{L^2}}$$

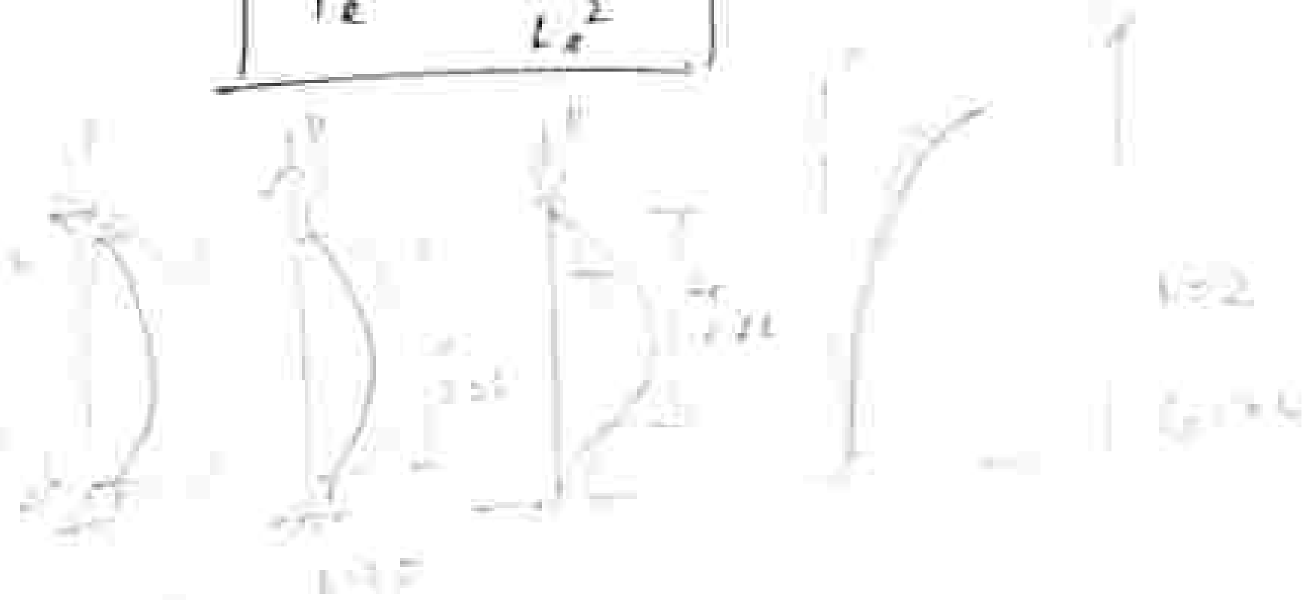
(d) One end fixed and one end free

$$\boxed{L_e = 2L}$$

$$\boxed{P_e = \frac{\pi^2 EI}{4L^2}}$$

The effective length of a given column with given end conditions is the length of an equivalent column of the same material and cross-section with hinged ends, and having the value of the crippling load equal to that of the given column.

$$\boxed{P_e = \frac{\pi^2 EI}{L_e^2}}$$



Torsion of Circular Shafts

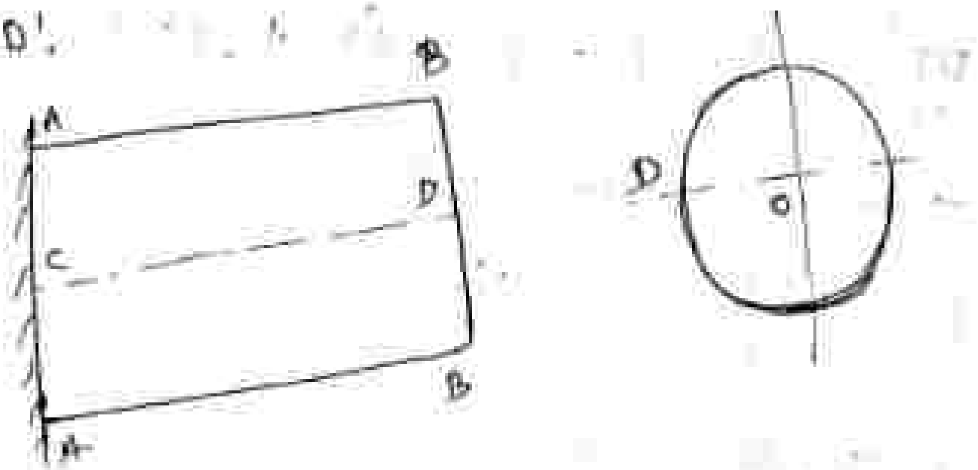
In workshops and factories, a turning force is always applied to transmit energy by rotation. The turning force is applied either to the rim of a pulley, keyed to the shaft or any other suitable point at some distance from the axis of the shaft. The product of this turning force and the axis of the shaft is known as torque, turning moment or twisting moment. And the shaft is subjected to torsion. Due to this torque, every cross-section of the shaft is subjected to some shear ~~force~~ stress.

OR

A shaft is said to be in torsion, when equal and opposite torques are applied at the two ends of the shaft. The torque is equal to the product of the force applied (tangentially to the ends of a shaft) and radius of the shaft. Due to the application of torques at the two ends, the shaft is subjected to a twisting moment. This causes the shear stress and shear strain in the material of the shaft.

Shear stress in circular shaft due to Torsion

Consider a shaft fixed at one end AA and free at the end BB as shown in fig. Let CD is any line on the outer surface of the shaft. Now let the shaft is subjected to a torque T at the end BB. As a result of this torque T, the shaft at the end BB will rotate c.w and every cross-section of the shaft will be subjected to shear stresses the point D will shifted to D' & hence line CD will be deflected to CD'. The line OD will be shifted to OD'.



Let R = Radius of shaft

L = Length "

T = Torque applied at the end BB

τ = shear stress induced at the surface of the shaft due to T

C, G = Modulus of rigidity of the material

ϕ = $\angle DCD'$ also equal to shear strain

θ = $\angle DOD'$ & is also called angle of twist.

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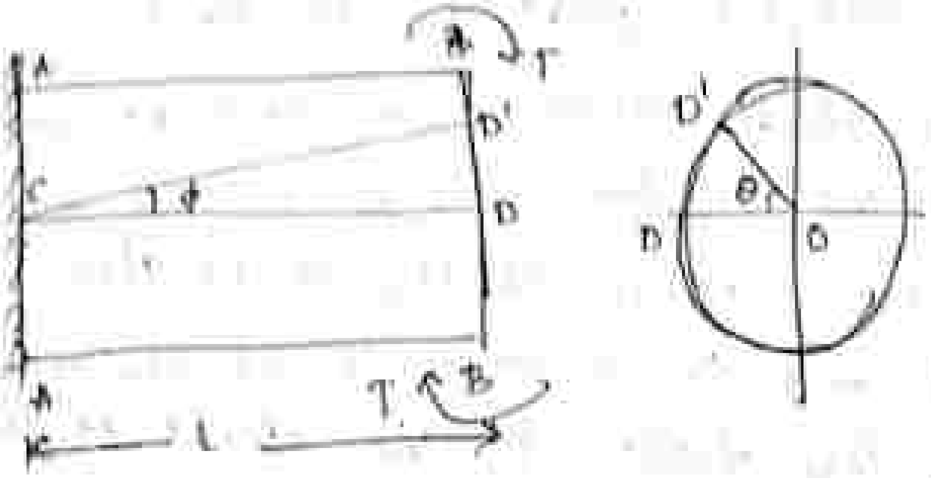

Now distortion at the outer surface

= Distortion per unit length

$$= \frac{\text{Distortion at the outer surface}}{\text{Length of shaft}} = \frac{DD'}{L}$$

$$= \frac{DD'}{CD} = \tan\phi = \phi \quad \text{—(i)} \quad \left(\tan\phi = \phi \text{ for small } \phi\right)$$

∴ shear strain at outer surface,

$$\phi = \frac{DD'}{L} \quad \text{—(i)}$$

Now, $DD' = OD \times \theta = R\theta$

Substituting the value of DD' in eqn (i),

shear strain at outer surface, $\phi = \frac{R \times \theta}{L}$

Now

$$G = \frac{\text{shear stress induced}}{\text{shear strain produced}}$$

$$= \frac{\text{shear stress at the outer surface}}{\text{shear strain at outer surface}}$$

$$= \frac{\tau}{\left(\frac{R \times \theta}{L}\right)} = \frac{\tau \times L}{R\theta} =$$

$$\therefore \boxed{\frac{G\theta}{L} = \frac{\tau}{R}} \quad \text{or} \quad \tau = \frac{R \times G \times \theta}{L}$$

Now for a given shaft subjected to Torque T, the value of G, θ, L are constant.

Hence $\tau \propto R$ or $\frac{\tau}{R}$ = constant

If q is the shear stress induced at a radius 'r' from centre of the shaft,

then, $\frac{\tau}{R} = \frac{q}{r}$

But, $\frac{\tau}{R} = \frac{G\theta}{L}$

$$\therefore \boxed{\frac{\tau}{R} = \frac{G\theta}{L} = \frac{q}{r}} \quad \text{or} \quad \boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}}$$

shear stress at any point is proportional to the distance of the point from the axis of the shaft. Hence shear stress is maximum at the outer surface and shear stress is zero at the axis of the shaft.

Assumptions for shear [illegible] subjected to torsion.

① The material of the shaft is uniform throughout.

② The twisting along the shaft is uniform.

③ Normal cross section of the shaft, which were plane and circular before the twist, remains plane & circular even after the twist.

④ The shaft is uniform circular section throughout.

⑤ All radii which are straight before twist remain straight after twist.

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